Instructions: Work on the problems in groups. Narrative adapted from *Khan Academy*.

1. We have added, subtracted, and multiplied polynomials. Now we will divide them. Let's recall how to divide numbers. If we were dividing 21 by 7, we could remember our times tables, or see that this is the same as simplifying the fraction $\frac{3\times 7}{3}$. From here, we could cancel the common factor of 3 and be left with the answer, 7.

   The same is true for something more complicated. Let's try to divide $3 \times 7 + 3 \times 4$, below. We are writing 33 in this "weird" way as it will be helpful for polynomials. Below are two equivalent methods; factoring on the left, splitting the fraction at each $+$ on the right:

$$\frac{3 \times 7 + 3 \times 4}{3} = \frac{\cancel{3}(7+4)}{\cancel{3}} \qquad \frac{3 \times 7 + 3 \times 4}{3} = \frac{\cancel{3}(7)}{\cancel{3}} + \frac{\cancel{3}(4)}{\cancel{3}}$$

   Now let's use the above method to divide polynomials. If we have $\frac{x(2x^2+x+1)}{x}$, there is a common factor in the numerator and the denominator, x, like there was a 3 before. Cancelling x leaves us with $2x^2+x+1$. This is dividing a polynomial by x. If we weren't given the factored form, we could do the division ourselves in either of the two ways above:

$$\frac{x^4 - 3x^2 + 2x}{x} = \frac{\cancel{x}(x^3 - 3x + 2)}{\cancel{x}} \qquad \frac{x^4 - 3x^2 + 2x}{x} = \frac{\cancel{x}(x^3)}{\cancel{x}} + \frac{\cancel{x}(-3x)}{\cancel{x}} + \frac{\cancel{x}(2)}{\cancel{x}}$$

   Use either of the above methods to divide:

   (a) $(4x^3 + 2x^2 + x) \div x$

   (b) $\frac{-5x^4+2x^2+8x}{x}$

2. If we were dividing $23 \div 7$, we don't get a whole number; we are left with 3 and a **remainder** of 2. Written another way, $3+2/7$. We could write the above work $\frac{23}{7} = \frac{21+2}{7} = \frac{3\times 7}{7} + \frac{2}{7} = 3 + \frac{2}{7}$ We do the same thing with polynomials:

$$\frac{5x^4 + 3x^2 + 4}{x} = \frac{5x^4}{x} + \frac{3x^2}{x} + \frac{4}{x} = 5x^3 + 3x + \frac{4}{x}$$

   In this case, the remainder of 4 can't be divided by x evenly, so we leave $4/x$ at the end. We have a quotient $p(x) = 5x^3 + 3x$ and a remainder $k = 4$.

   Find the quotient and remainder:

   (a) $\frac{4x^3-3x+1}{x}$

   (b) $(-5x^4 - x^2 - 8) \div x$

3. We can divide by more than just x. For example, we can divide $\frac{x^2-9}{x+3}$. Our first step should always be to try to factor and cancel, like in question 2:

$$\frac{x^2 - 9}{x+3} = \frac{(x-3)\cancel{(x+3)}}{\cancel{x+3}} = (x-3)$$

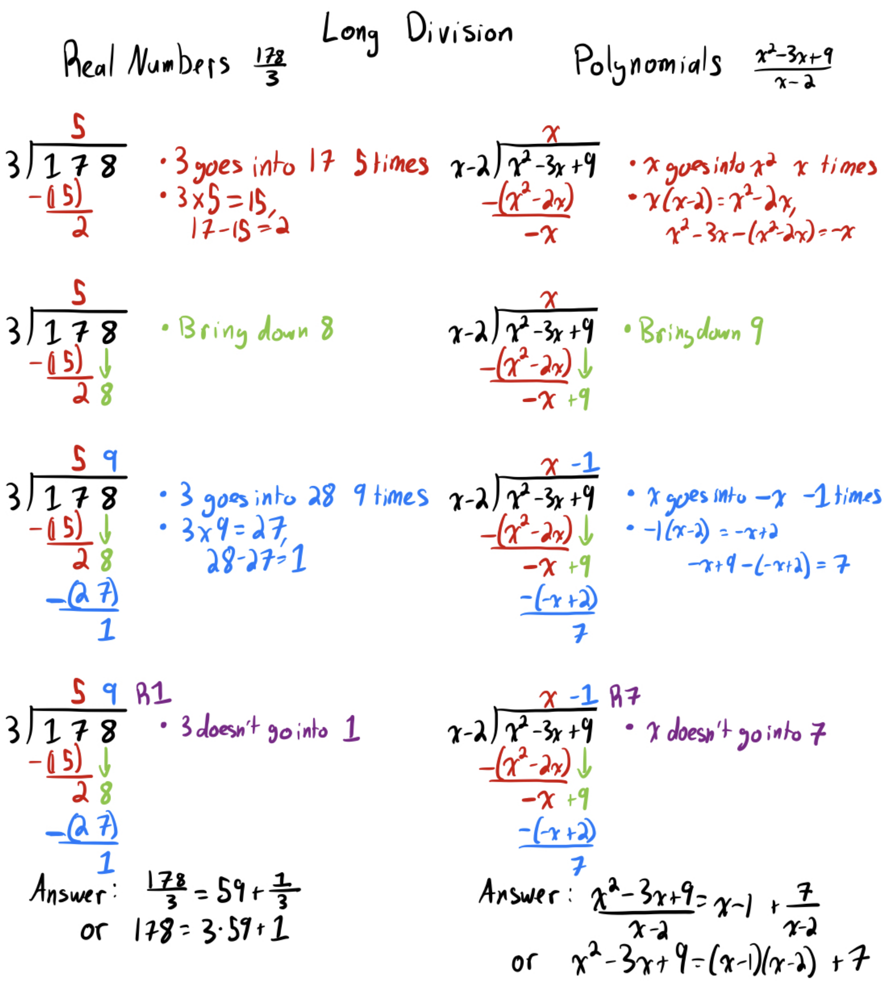
   (a) Divide $\frac{x^2-5x+4}{x-1}$

   (b) Divide $\frac{x^2+8x+16}{x+4}$

4. What if we want to divide something that doesn't factor nicely, like $178/3$ or $\frac{x^2-3x+9}{x-2}$? You should be able to check that these don't factor like previous questions. One technique is to try and write the numerator as something that does factor plus something that doesn't. Since is $178 = 177 + 1$ or $x^2 - 3x + 9 = x^2 - 3x + 2 + 7$, we could write the numerators as

$$178 = 59 \times 3 + 1 \qquad (x^2 - 3x + 2) + 7 = (x - 2)(x - 1) + 7$$

Then our problem can be solved using the remainder method as in question 3, leaving us with 59 or $x - 1 + \frac{7}{x-2}$. However, finding that "factor-able" part is often difficult, which is where long division comes in. Remember how we do long division for real numbers, on the left, and translate that to polynomials, on the right.



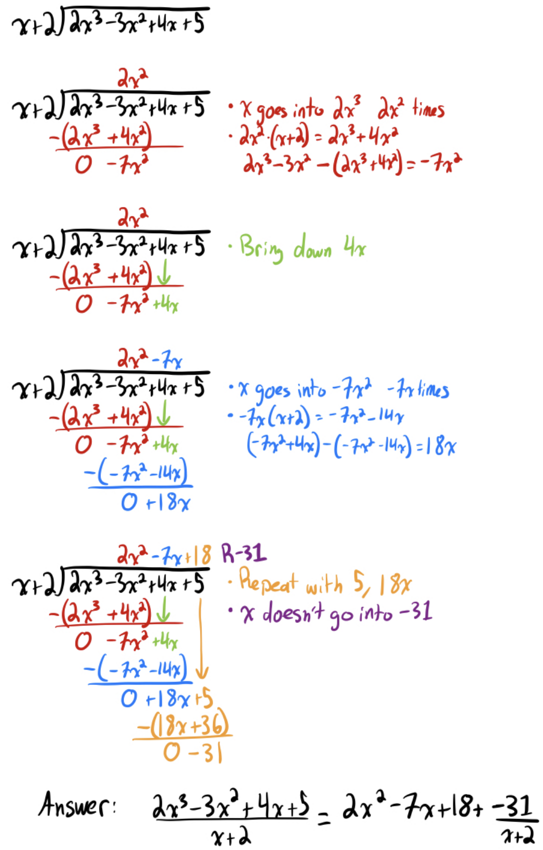
You should understand every step in that procedure. Note that whenever there is a power of x missing, like $2x^2 + 3$ has no x^1 term, you **should always write the missing term with a** 0 **coefficient**, $2x^2 + 0x + 3$. This will keep your powers aligned.

Divide two more quadratics to find the quotient and remainder:

(a) $\frac{x^2+3}{x-2}$

(b) $\frac{x^2+9x+14}{x+4}$

5. We can divide more than quadratics. For example, if we were to divide $2x^3 - 3x^2 + 4x + 5$ by $x + 2$ using the long division algorithm, it would look like this:



This means that $2x^3 - 3x^2 + 4x + 5 = (x + 2)(2x^2 - 7x + 18) - 31$

(a) $(5x^2 + 3x - 2) \div (x + 1)$

(b) $\frac{6x^3+11x^2-31x+15}{3x-2}$

6. When we divide $\frac{23}{7} = 3 + \frac{2}{7}$, that is the same thing as writing $23 = 3 \times 7 + 2$. The same is true of polynomials. If $\frac{p(x)}{q(x)} = h(x) + \frac{R(x)}{q(x)}$, then $p(x) = h(x) \times q(x) + R(x)$. **If the remainder, $R(x)$, is** 0, then $p(x)$ factors into $h(x) \times q(x)$.

Now if we divide by the polynomial $q(x) = x - a$, then when we plug in $x = a$, we get

$$p(a) = h(a) \times (a - a) + R = 0 + R$$

So to find $p(a)$, we can either plug a into the polynomial and evaluate, or divide the polynomial by $x - a$ and take the remainder.

(a) $(x - 4)$ is a factor of $p(x) = 3x^3 - 20x^2 + 37x - 20$, so the remainder is 0.
   i. Divide $p(x)$ by $(x - 4)$. Your answer, $h(x)$ tells you that $p(x) = (x - 4)h(x)$
   ii. Factor $h(x)$, a quadratic polynomial.
   iii. List all of the roots of $p(x)$. (The question tells you one of them. The other's should come from $h(x)$, right?)

(b) If $q(x) = 2x^5 + 3x^4 - 2x + 3$, use the formula above to find $q(3)$.