Day 10

Last Time: Perpendicular & Parallel Lines

Quadratics: $ax^2+bx+c = a(x-h)^2+k = f(x)$

$a=a$

$h=\frac{-b}{2a}$ $\quad k=f(h)$

vertex: (h,k) $\quad$ min/max occurs at $x=h$

value $y=k$

End behavior of Power Functions & Polynomials

Get Started: 1) (Itempool) Find the value (y-coord) of the minimum of the quadratic $f(x)=2x^2-6x+7$

2) Which of the following are polynomials?

a) $f(x)=2x^3\cdot 3x+4$ is a poly

b) $g(x)=-x(x^2-4)$ $=-x^3+4x$ is a poly

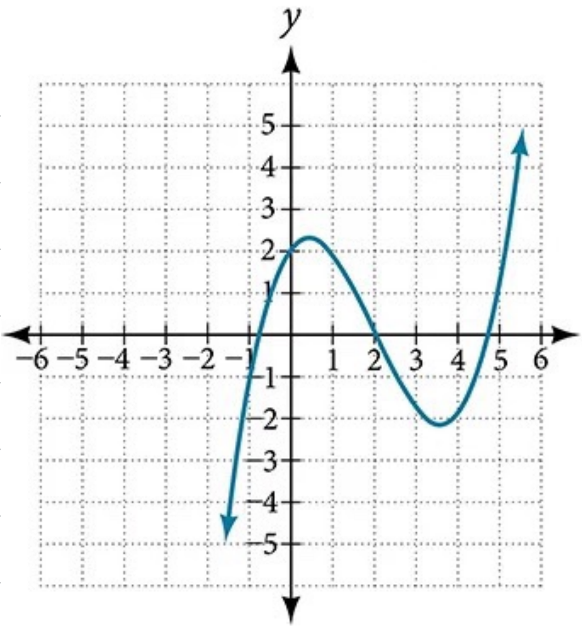
c) $h(x)=5\sqrt{x+2}$ ← not a poly

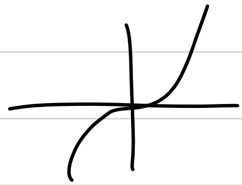
1): x-coord $h=-\frac{b}{2a}=-\frac{-6}{2\cdot 2}=\frac{6}{4}=\frac{3}{2}$

y-coord $f(\frac{3}{2})=2(\frac{3}{2})^2-6(\frac{3}{2})+7$

$=\frac{9}{2}-9+7=\frac{9}{2}-2=\frac{9-4}{2}=\frac{5}{2}$

ends go in
diff. direction
degree is odd
leading coeff

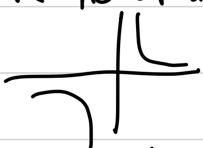




$x \to \infty \qquad f(x) \to \infty$

$x \to -\infty \qquad f(x) \to -\infty$

Graphs of polynomials are smooth and continuous
there are no breaks or sharp corners



$1/x$

$|x|$

Finding roots of Polynomials
where does the graph cross x-axis?
where does $y=0$

Ex: Roots of $f(x) = x^6 - 3x^4 + 2x^2$

$$0 = x^6 - 3x^4 + 2x^2$$
$$= x^2(x^4 - 3x^2 + 2)$$
$$= x^2 \; (x^2 - 2)(x^2 - 1)$$

$x^2 = 0$ $\quad$ $x^2 = 2$ $\quad$ $x^2 = 1$

$x = 0$ $\quad$ $x = \pm\sqrt{2}$ $\quad$ $x = \pm 1$

Ex: $g(x) = (x-2)^2(2x+3)$

$(x-2)^2 = 0$ $\quad$ $2x+3 = 0$

$x - 2 = 0$ $\quad$ $x = -\frac{3}{2}$

$x = 2$

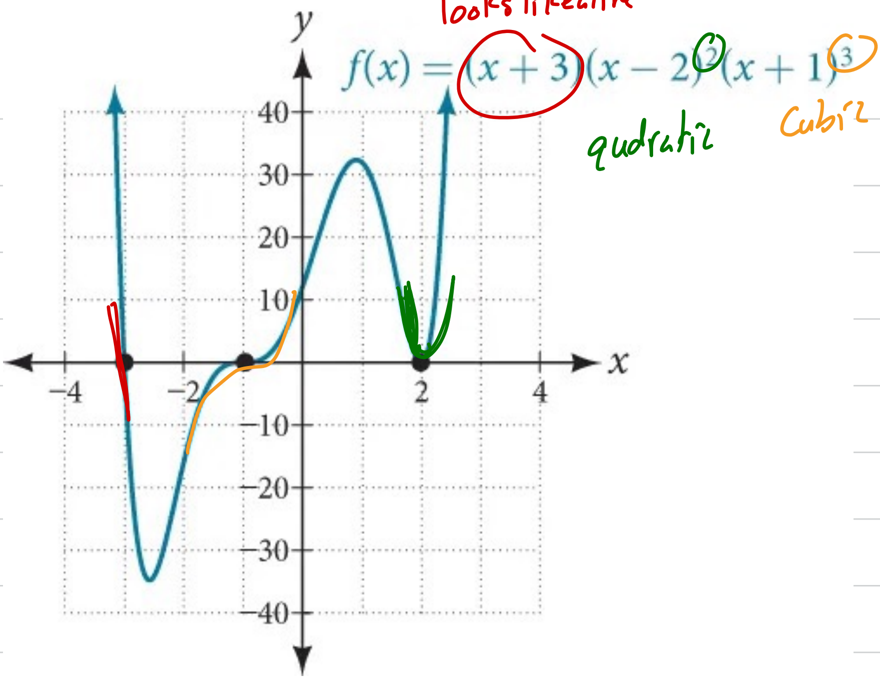
y-int is where the graph crosses the y-axis

$x = 0$

$$g(0) = (0-2)^2(0+3)$$
$$= 4 \cdot 3 = 12$$

$(x-2)^2$

$(x-2)(x-2)$

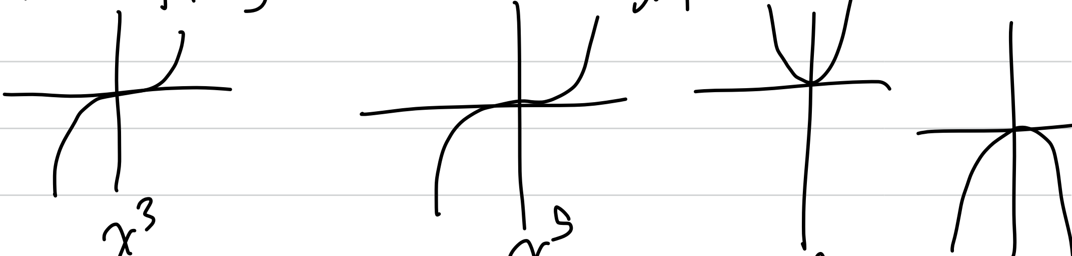


multiplicity of the roots

If a polynomial factors to contain $(x-h)^p$

then $x = h$ is a root of multiplicity p

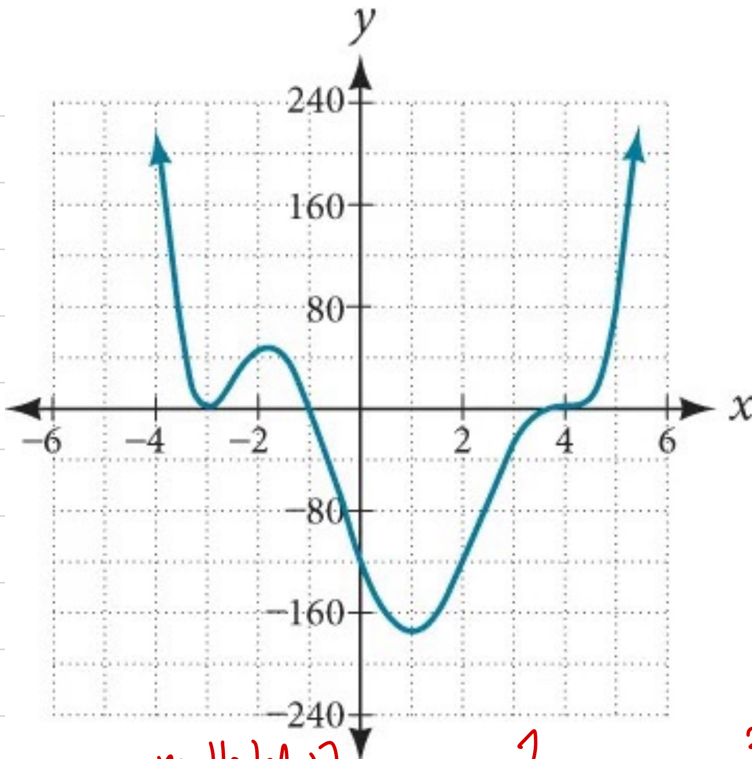
the multiplicity determines how the graph acts at that root



x^3 x^5 x^2 x^4

odd → through x-axis

even → bounces off x-axis



multiplicity: 2 $\qquad\qquad$ 1 $\qquad\qquad$ 3

roots at $x = -3 \qquad x = -1 \qquad x = 4$

degree is even, leading coeff pos.

$$f(x) = a(x+3)^2 (x+1)^1 (x-4)^3$$

The number of roots with multiplicity must be less than or equal to the degree of the polynomial

The number of local mins/maxs must be $\leq$ deg -1

(turning points)

Ex: $f(x) = -2(x+3)^2(x-5) = -2(x^2+6x+9)(x-5)$

$= -2(x^2 x$

roots

$x = -3$ mult. 2

$x = 5$ mult 1

$f(0) = -2(0+3)^2(0-5)$ y-int

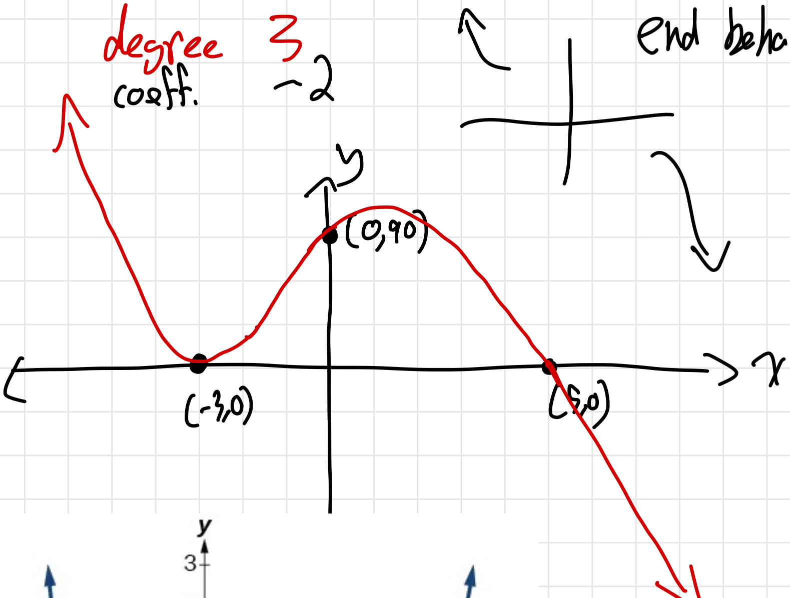
$= -2(9)(-5)$

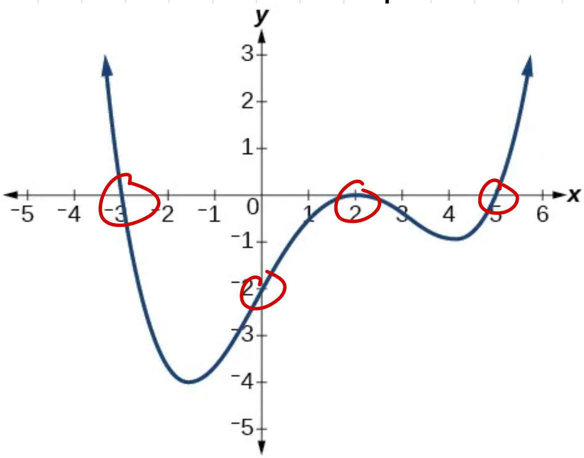
$= 10 \cdot 9 = 90$

degree 3

coeff. -2

end behavior





$f(x) = a(x+3)^1(x-2)^2(x-5)^1$

$f(0) = -2$

$-2 = a(0+3)(0-2)^2(0-5)$

$= a\ 3 \cdot 4(-5)$

$-2 = -60a \rightarrow a = +\frac{1}{30}$

$f(x) = \frac{1}{30}(x+3)(x-2)^2(x-5)$