Instructions:

- Time: 75 minutes.
- Write in pen or pencil.
- No notes, textbooks, phones, calculators, or other electronic devices.
- Show your work and justify your answers. You will not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. Clearly circle or label your final answers.

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<th>QUESTION</th>
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1. (15 points) Consider the function $f$ with

$$f(x) = \frac{\ln x}{x^2}, \quad f'(x) = \frac{1 - 2 \ln x}{x^3}, \quad f''(x) = \frac{6 \ln x - 5}{x^4}.$$ 

(a) Find the domain of $f$.

(b) List all $x$- and $y$-intercepts of $f$.

(c) List all vertical asymptotes of $f$ or explain why none exist.
(d) List all horizontal asymptotes of $f$ or explain why none exist.

(e) On what intervals is $f(x)$ increasing? decreasing?

(f) On what intervals is $f(x)$ concave up? concave down?
2. (10 points) Sketch the graph of a function $g(x)$ that satisfies the following properties:

- $g$ is continuous at all points of its domain
- $x$-intercepts: $-3, 2$
- $y$-intercept: $-2$
- $\lim_{x \to 3^-} g(x) = \infty$ and $\lim_{x \to 3^+} g(x) = \infty$
- $\lim_{x \to -\infty} g(x) = 5$ and $\lim_{x \to \infty} g(x) = 5$
- increasing on $(-\infty, -6) \cup (-1, 3)$
- decreasing on $(-6, -1) \cup (3, \infty)$
- concave up on $(-\infty, -7) \cup (-3, 3) \cup (3, \infty)$
- concave down on $(-7, -3)$
3. (10 points) Find the area of the largest rectangle which has two vertices on the $x$-axis and two vertices on the graph of the function $y = 8 - x^2$ with $-\sqrt{8} \leq x \leq \sqrt{8}$.
4. (15 points)

(a) Estimate the area under the graph \( y = \sqrt{x} \) between \( x = 0 \) and \( x = 8 \) by using a Riemann sum with four intervals of equal width and right endpoints.

(b) Find a number \( U \) such that
\[
\int_{\pi/2}^{\pi} e^{2\sin x} \, dx \leq U.
\]
Use the comparison properties of the integral to justify your answer.
5. (15 points)

(a) Evaluate
\[ \frac{d}{dx} \int_x^{e^x} \frac{t^2 + 1}{\sqrt{t} + 1} \, dt \]

(b) Evaluate
\[ \int_1^5 \frac{x^3 e^x + \sqrt{x} e^{2x}}{\sqrt{x} e^x} \, dx \]
6. (10 points) A particle is moving with the given acceleration, velocity, and position data. Find the position function $s(t)$ of the particle.

\[ a(t) = 3 \cos(t) - 2 \sin(t), \quad v(\pi) = 2, \quad s(0) = 3. \]