MATH 142
MIDTERM EXAM I
February 20, 2003

NAME (please print legibly): _____________________________________________
Your University ID Number: ___________________________________________
Circle your Instructor’s Name along with the Lecture Time:
Caulk (9 o’clock)  Knightly (10 o’clock)  Moustafaev (2 o’clock)  Qiu (2 o’clock)

• No calculators are allowed on this exam.

• Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.

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1. (15 pts) Let \( f(x) = \frac{1 + x^2}{1 - x^2} \). Then \( f'(x) = \frac{4x}{(1 - x^2)^2} \) and \( f''(x) = \frac{8x^2 + 4}{(1 - x^2)^3} \).

(a) Find the domain of \( f \).

(Answer: ______________________)

(b) Find all horizontal and vertical asymptotes of \( f \).

(Answer: ______________________)
...Continue with \( f(x) = \frac{1 + x^2}{1 - x^2} \), \( f'(x) = \frac{4x}{(1 - x^2)^2} \) and \( f''(x) = \frac{8x^2 + 4}{(1 - x^2)^3} \).

(c) Find the intervals on which \( f \) is increasing and the intervals on which \( f \) is decreasing.

**Answer:** Increasing on:_________________ Decreasing on:_________________

(d) Find all local extrema (if any) for \( f \).

**Answer:** Local Max(’s):_________________ Local Min(’s):_________________

(e) Find the intervals on which \( f \) is concave up and those on which it is concave down.

**Answer:** CU on:_________________ CD on:_________________
2. (10 pts) Using the information from the previous problem, sketch the graph of

\[ y = \frac{1 + x^2}{1 - x^2} \]

on the axes below. Label the coords of all max/mins and intercepts, and include the equations for all asymptotes. (On the axes below, 1 mark = 1 unit.)
3. (10 pts) The width of a certain rectangle is four times the reciprocal of its length. What is the smallest possible value for the perimeter?

Your answer should be the perimeter, not the width!
4. (5 pts) Suppose we are trying to solve \( x^3 - x + 3 = 0 \) using Newton’s method. If our first guess is \( x = 1 \), what will our second guess be?
5. **(12 pts)** A ball is thrown straight up from the top of an 80 ft building with an initial velocity of 64 ft/sec. (The acceleration due to gravity is $-32 \text{ ft/sec}^2$.)

(a) Find the formula for the velocity $v(t)$ at time $t$.  

```plaintext
ANSWER: ____________________________
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(b) Find the formula for the height $h(t)$ at time $t$.  

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ANSWER: ____________________________
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(c) When does the ball reach its highest point?  

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ANSWER: ____________________________
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(d) When does the ball land on the ground?  

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ANSWER: ____________________________
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6. (12 pts) Find the most general antiderivatives for the following:

(a) \( f(x) = \sqrt{x} + 5x^2 - \frac{7}{x} \)

ANSWER: ______________________

(b) \( f(x) = \frac{5}{1+x^2} \)

ANSWER: ______________________

(c) \( f(x) = \sin(2x) + e^{-x} \)

ANSWER: ______________________
7. (12 pts) Approximate the value of the integral
\[
\int_{1}^{7} (x^2 + 2) \, dx
\]
using the Riemann sum with \( n = 3 \) rectangles, using right endpoints. You don’t have to simplify your answer.
8. **(12 pts)** Evaluate $\int_{-1}^{2} |x| \, dx$ by drawing a picture and computing the appropriate area.
9. (12 pts) Find a point on the parabola \( y = x^2 - 1 \) whose distance from the point \((0, 3)\) is as small as possible.