MATH 142
Midterm 2
November 5, 2002

NAME (please print legibly): ________________________________
Your University ID Number: ________________________________
Circle your Instructor’s Name along with the Lecture Time:

Zokhrab Moustafaev (MWF 9:00 - 9:50)  Carl Mueller (MW 2:25 - 4:40)

• No calculators are allowed on this exam.
• Please show all your work. You may use back pages if necessary. You may
  not receive full credit for a correct answer if there is no work shown.
• Please put your final answers in the spaces provided.

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<th>QUESTION</th>
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1. (30 pts) Solve the following integrals.

(a) (10 pts)
\[ \int 3 \sin(2x) \, dx \]

(b) (10)
\[ \int \frac{2}{\sqrt{x}} \, dx \]

(c) (10)
\[ \int_{-4}^{-2} \frac{2 + x}{5x} \, dx \]
2. (26 pts)

(a) (13 pts) Find
\[ \frac{d}{dx} \int_0^{\sqrt{x}} (1 + t^4)dt \]

(b) (13 pts) Find
\[ \frac{d}{dx} \int_0^{x^2} \sin^4(t) \, dt \]
3. (15 pts) Suppose that you keep track of the rainfall, in inches per hour, for Rochester. Time is measured in hours. It is now time 0. At time $t$, it is raining at $(t^2 + t)/10,000$ inches per hour. Find the amount of rainfall over a 3-day period, starting now.

**Hint:** How many hours are in 3 days?
4. (39 pts) Solve the following integrals.

(a) (13 pts)
\[
\int \frac{\sin(\ln(x))}{x} \, dx
\]

(b) (13 pts)
\[
\int_{0}^{\pi/4} \cos(2x)e^{\sin(2x)} \, dx
\]
(c) (13 pts)

\[ \int_{0}^{1} \frac{e^x}{e^x + 1} \, dx \]
5. (15 pts) Find the area between the curves

\[
\begin{align*}
  y &= 3x + 3 \\
  y &= 3 - x^2
\end{align*}
\]

between \(x = 0\) and \(x = 1\).
6. (15 pts) Find the area between the curves

\[ y = x^2 - 1 \]
\[ y = x + 1 \]

Hint: Find the points at which the curves intersect.
7. (15 pts) Find the volume of the solid obtained by rotating the region bounded by the given curves, about the $x$-axis.

$$y = x^2$$
$$y = 2x$$
8. **(15 pts)** Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the $y$-axis.

**WARNING** Unless you use the shell method, you will not get full credit.

\[ y = e^{x^2} \]
\[ y = 0 \]
\[ x = 0 \]
\[ x = 4 \]
9. (15 pts) A spring at rest has length of 1 meter. Assuming that the spring constant $k$ equals 10 Newtons per meter squared. Calculate the work required to stretch the spring so as to increase its length to 3 meters.
10. (15 pts) Find the average value of \( f(x) = x\sqrt{1 + x^2} \) over \([0, 4]\).