Math 430 Problem Set #11 Due 11/17/21

1. Show that $|\operatorname{Cl}(\mathbb{Z}[\frac{1+\sqrt{29}}{2}])| = 1$.

2. Show that $|\operatorname{Cl}(\mathbb{Z}[\frac{1+\sqrt{-19}}{2}])| = 1$.

3. Use your result from #3 to find all the integers solutions x and y to the equation $x^2 + 19 = y^3$, the equation we studied the first day of class.

4. Let $d < -1$ be a squarefree number congruent to 3 (mod 4). Show that the prime ideal $\mathfrak{p} \subseteq \mathbb{Z}[\sqrt{d}]$ with $\mathfrak{p} \cap \mathbb{Z} = 2$ has order 2 in the class group $\operatorname{Cl}(\mathbb{Z}[\sqrt{d}])$.

5. Calculate the class group of $\mathbb{Z}[\sqrt{-5}]$.

6. Let $d < -1$ be composite and squarefree. Let

$$\omega = \begin{cases} \sqrt{d} & : \quad d \equiv 2, 3 \pmod 4 \\ \frac{1+\sqrt{d}}{2} & : \quad d \equiv 1 \pmod 4 \end{cases}$$

Show that $|\operatorname{Cl}(\mathbb{Z}[\omega])| \neq 1$. [Hint: Look at the prime in $\mathbb{Z}[\omega]$ lying over the smallest positive prime factor of d.]