Math 531 Problem Set #10 Due 11/10/04

1. Let $m > 2$ be an integer that is not a prime power and let ξ_m be a primitive m-th root of unity. Show that $N_{\mathbb{Q}(\xi_m)/\mathbb{Q}}(1 - \xi_m) = \pm 1$.

2. Let L be a degree n field extension of $\mathbb{Q}$. Let $B \subset L$ be a ring that is integral over $\mathbb{Z}$ and has field of fractions L. Let $\sigma_1, \ldots, \sigma_n$ be the n distinct embeddings $\sigma : L \longrightarrow \mathbb{C}$. Show that for any basis $w_1, \ldots, w_n$ for B as an A-module, we have

$$\Delta(B/\mathbb{Z}) = (\det[\sigma_i(w_j)])^2 .$$

[Hint: Multiply $[\sigma_i(w_j)]$ by its transpose and use the fact (that you should prove) that $\mathrm{T}_{L/K}(y) = \sigma_1(y) + \cdots + \sigma_n(y)$ for any $y \in L$.]

3-5 Ex. 1,3,4 from p. 62.

6. p. 65, Ex. 1.

7. Let m be a positive integer and let ξ_m be a primitive m-th root of unity. Show that $\mathrm{T}_{\mathbb{Q}(\xi_m)/\mathbb{Q}}(\xi_m) = \mu(m)$, where μ is the Mobius function defined in Ex. 1, p. 57.

8. Let $\mathcal{O}_L$ be the integral closure of $\mathbb{Z}$ in some finite field extension L of $\mathbb{Q}$. Let M be any positive real number. Show that there are finitely many ideals $I \subset \mathcal{O}_L$ such that $\mathrm{N}_{\mathcal{O}_L/\mathbb{Z}}(I) \leq M$.