MATH 265H, FALL 2022, HOMEWORK #8

ALEX IOSEVICH

1. Problems not in the book

Problem #1: Let $N(R)$ denote the number of points with integer coordinates contained in the disk of radius R (large) centered at the origin, i.e

$$N(R) = \sum_{k \in \mathbb{Z}^2} \chi_{RD}(k),$$

where D is the disk of radius 1, $k = (k_1, k_2)$,

$$RD = \{Rx : x \in D\},$$

and $\chi_{RD}(x)$ is equal to 1 if $x \in RD$, and 0 otherwise.

Prove that there exists a finite positive constant C, independent of R, such that

$$\left| N(R) - \pi R^2 \right| \leq CR.$$

Problem #2: Consider the series

$$\sum_{n=1}^{\infty} \frac{a_i}{n},$$

where a_i is chosen randomly by flipping a fair coin, with heads producing $+1$ and tails -1. Prove that the expected value of this sum is finite.

Note: I have no problem with you looking this up and treating this exercise as a library science project, as long as you learn the proof and understand all the underlying concepts. You may run into definition and ideas you have not seen before, so just do the best you can.

2. Problems from the book

Chapter 4, problems 1,2,3,4,8,9