MATH 265H, FALL 2022, HOMEWORK #7

ALEX IOSEVICH

1. Problems not from the book

Problem #1: Let $k = (k_1, k_2) \in \mathbb{N} \times \mathbb{N}$. Let n be a positive integer. Find the values of p for which the sequence

$$\sum_{1 \le ||k|| \le n} \frac{1}{||k||^p}$$

converges as $n \to \infty$. Here $||k|| = \sqrt{k_1^2 + k_2^2}$. Prove all your assertions.

Hint: The book proves this in the one-dimensional case. The point of this problem is to understand what the one-dimensional proof really says.

Problem #2: Let $f(n)$ be a function such that for every k, $k = 0, 1, 2, \dots,$ exactly 9 of the values of f at $10k, 10k+1, \dots, 10(k+1)-1$ are equal to 1 and the remaining value is equal to -1. Is it true that for any such function f,

$$\sum_{n=1}^{\infty} \frac{f(n)}{n}$$

converges? Prove all your assertions.

Hint: You may want to revisit the proof that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges and think about what it really says.

2. Problems from the book

Page 78, problems 14, 17, 18, 25