MATH265H, FALL2022, HOMEWORK #6

ALEX IOSEVICH

1. Problems not from the book

Problem #1: Let S be a finite subset of $\mathbb{R}^3$ of size N. Let $x = (x_1, x_2, x_3)$, and define

$$\pi_1(x) = (x_2, x_3), \;\; \pi_2(x) = (x_1, x_3), \;\; \pi_3(x) = (x_1, x_2).$$

Prove that

$$N^2 \leq \#\pi_1(S) \cdot \#\pi_2(S) \cdot \#\pi_3(S),$$

where $\#\pi_j(S)$ denotes the number of elements in

$$\pi_j(S) = \{\pi_j(x) : x \in S\}.$$

Hint: Prove that

$$\chi_S(x) \leq \chi_{\pi_1(S)}(x_2, x_3) \cdot \chi_{\pi_2(S)}(x_1, x_3) \cdot \chi_{\pi_3(S)}(x_1, x_2),$$

where given a set A, $\chi_A(x) = 1$ if $x \in A$ and 0 otherwise. After establishing this inequality (don't work too hard...), apply Cauchy-Schwarz in the right way to complete the proof.

2. Problems from the book

Page 78, problems 13, 16, 19, 20, 21, 22