# MATH265H, FALL 2022, HOMEWORK #5

ALEX IOSEVICH

## 1. Problems not in the book

**Problem:** Let $A$ be an $n$ by $n$ matrix with the following properties:

- i) $A_{ij}$ is equal to 1 or 0.
- ii) Suppose that $A_{ij}A_{ij'} = 1$ for some $i, j, j'$ with $j \neq j'$. Then $A_{i'j}A_{i'j'} = 0$ for any $i' \neq i$.

Prove that the number of 1's in $A$ does not exceed $10n^{\frac{3}{2}}$.

**Hint:** You are trying to estimate $\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}$. Consider

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{n} A_{ij}\right)\cdot 1.$$

Let $a_i = \sum_{j=1}^{n} A_{ij}$, $b_i = 1$, and apply the Cauchy-Schwartz inequality we proved in class, i.e

$$\sum_{i=1}^{n} a_i b_i \leq \left(\sum_{i=1}^{n} a_i^2\right)^{\frac{1}{2}} \left(\sum_{i=1}^{n} b_i^2\right)^{\frac{1}{2}}.$$

## 2. Problems from the book

Page 78, problems 5, 6, 7, 9, 10, 11