MATH 173, FALL 2022, HOMEWORK #8

ALEX IOSEVICH

1. Problems not in the book

Problem #1: Let $p > 1$ and let V be the set consisting of continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$\lim_{N\to\infty} \int_{-N}^{N} \frac{|f(x)|^p}{1+x^2} dx$$

exists and is finite. The integral is the usual Riemann integral.

i) Give an example of $f \in V$ such that f is unbounded, i.e there **does not** exist $M > 0$ such that $|f(x)| \leq M$.

ii) Prove that V is a vector space over the real numbers.

iii) Prove that V is infinite dimensional.

2. Problems from the book

Section 3.1, problems 1,4,7, 9,11

Section 3.2 problems 1,2,4,9,11