# MATH 173, FALL 2022, HOMEWORK \#6 

## ALEX IOSEVICH

## 1. Problems not in the book

Problem \#1: Let $f$ be a twice continuous differentiable function on $\mathbb{R}$. Suppose that $f^{\prime \prime}(x) \geq 0$ for all $x \in \mathbb{R}$.
i) Prove that for $t \in[0,1]$ and $x<y$ real numbers,

$$
f((1-t) x+t y) \leq(1-t) f(x)+t f(y)
$$

ii) Generalize part i) and show that if $0 \leq a_{i} \leq 1, \sum_{i=1}^{n} a_{i}=1$, and $x_{1}, \ldots, x_{n}$ real numbers, then

$$
f\left(\sum_{i=1}^{n} a_{i} x_{i}\right) \leq \sum_{i=1}^{n} a_{i} f\left(x_{i}\right) .
$$

iii) Use ii) to prove that if $c_{1}, c_{2}, \ldots, c_{n}$ are positive real numbers, then

$$
\left(\prod_{j=1}^{n} c_{j}\right)^{\frac{1}{n}} \leq \frac{1}{n} \sum_{i=1}^{n} c_{i}
$$

Note: You solved part iii) on last week's homework, but I want you to solve it here using part ii), not using the induction argument outlined last week.
2. Problem from the book

Section 2.4, problems 1,2,3,5,6,7

