MATH 173, FALL 2022, HOMEWORK #6

ALEX IOSEVICH

1. PROBLEMS NOT IN THE BOOK

Problem #1: Let f be a twice continuous differentiable function on \mathbb{R} . Suppose that $f''(x) \ge 0$ for all $x \in \mathbb{R}$.

i) Prove that for $t \in [0, 1]$ and x < y real numbers,

$$f((1-t)x + ty) \le (1-t)f(x) + tf(y).$$

ii) Generalize part i) and show that if $0 \le a_i \le 1$, $\sum_{i=1}^n a_i = 1$, and x_1, \ldots, x_n real numbers, then

$$f\left(\sum_{i=1}^{n} a_i x_i\right) \le \sum_{i=1}^{n} a_i f(x_i).$$

iii) Use ii) to prove that if c_1, c_2, \ldots, c_n are positive real numbers, then

$$\left(\prod_{j=1}^n c_j\right)^{\frac{1}{n}} \le \frac{1}{n} \sum_{i=1}^n c_i.$$

Note: You solved part iii) on last week's homework, but I want you to solve it here using part ii), not using the induction argument outlined last week.

2. PROBLEM FROM THE BOOK

Section 2.4, problems 1,2,3,5,6,7