Abstract

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What is the greatest number of points that only determine one distinct triangle and what configurations of such points are possible? What if we consider n distinct triangles in d dimensions? It turns out we can construct unique answers to this question when $n = 1, 2$ for $d > 2$, with much of the work being done purely combinatorially without reference to dimension or, in fact, a metric at all. In future work, we are hopeful that the combinatorial framework developed in these proofs can be used to make some simplifications to the general problem. The development of these proofs takes us on a tour of Erdős problems, finite metric spaces, basic graph theory, high school geometry and an obscure enumerative combinatorics paper from the 1950s.