In class we proved that \((A \cup B) \setminus B \subseteq A\). Let’s go over the proof to make sure that the logic is clear. Suppose that \(X \in (A \cup B) \setminus B\). Let \(P\) be the statement that \(x \in A\), and \(Q\) the statement that \(x \in B\). Then the expression

\[(A \cup B) \setminus B\]

is encoded by

\[P \lor Q \land \neg Q.\]

The point that I did not make sufficiently clear in class is the following. The truth table for \(P\) is NOT the same as the truth table for \(P \lor Q \land \neg Q\), but this IS NOT THE POINT. The point is that \(P\) is TRUE whenever \(P \lor Q \land \neg Q\) is TRUE, which is all we need because we only need to conclude that \(x \in A\), which is encoded by \(P\).

Indeed, \(P \lor Q \land \neg Q\) is TRUE only when \(P\) is TRUE and \(Q\) is FALSE. Under these conditions \(P\) is TRUE and this all we need to know because out goal is to show that if \(x \in (A \cup B) \setminus B\), then \(x \in A\). This is precisely what we just did.

Let’s take a step back and try to understand this from the point of view of sets. The expression \(P\) and the expression \(P \lor Q \land \neg Q\) CANNOT have the same truth table because when \(P\) and \(Q\) are both true, \(x\) is contained in the intersection of \(A\) and \(B\). In particular, this means that \(x \in B\), which in incompatible with \(P \lor Q \land \neg Q\) where the possibility that \(x \in B\) is excluded by the condition \(\neg Q\).