Due Monday, September 13 at the beginning of class.

(1) Prove every prime of the form $3k + 1$ is also of the form $6k + 1$.

(2) Prove any positive integer of the form $3k + 2$ has a prime factor of the same form. Prove the same claim for positive integers of the form $4k + 3$ and $6k + 5$.

(3) If $\gcd(a, b) = p$, where p is prime, what are the possible values of $\gcd(a^2, b)$? $\gcd(a^3, b)$? $\gcd(a^2, b^3)$?

(4) Suppose $\gcd(a, p^2) = p$ and $\gcd(b, p^3) = p^2$, where p is prime. Evaluate $\gcd(ab, p^4)$ and $\gcd(a + b, p^4)$.

(5) Find an integer n such that $\frac{n}{2}$ is a square, $\frac{n}{3}$ is a cube, and $\frac{n}{5}$ is a fifth power.

(6) Suppose $a \mid bc$ and $\gcd(a, b) = 1$. Show $a \mid c$.