Due Wednesday, October 6 at the beginning of class. All chapter and exercise numbers refer to Silverman's *A Friendly Introduction to Number Theory*, 4th edition.

(1) Ex. 12.2

(2) Ex. 12.6

(3) Let S be the set of squares, i.e. $S = \{m^2 \mid m \in \mathbb{N}\}$.

   (a) Fix $n \in \mathbb{N}$. Give a good upper bound of $\#\{a \in S \mid a \leq n\}$.

   (b) Use your upper bound in (a) to show that S has *density zero*. That is, if $f(x) = \#\{a \in \mathbb{N} \mid a \leq x\}$, then

$$\lim_{x \to \infty} \frac{f(x)}{x} = 0.$$

(4) Ex. 13.3. Conclude that gaps between primes can be arbitrarily large.

(5) Ex. 15.1

(6) Ex. 15.3 (b) and (e). You may find the other parts of the problem helpful in noticing patterns, but you do not need to turn in solutions for them.

(7) Let p be a prime. Define the Mersenne number $M_p = 2^p - 1$ (not necessarily prime).

   (a) Suppose q is a prime divisor of M_p. Use the result of the first exercise in Problem Set 2 to show that $p \mid (q-1)$.

   (b) Use part (a) to give a proof different than Euclid's that there are infinitely many primes.