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On $BP_*\Omega(SU(n)/SO(n))$

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Abstract. Let $SU(n)$ be the n-th special unitary group, $SO(n)$ be the n-th special orthogonal group, and $SU(n)/SO(n)$ be the homogenous space. Let SU be the infinite special unitary group, SO be the infinite special orthogonal group, and SU/SO be the homogenous space. By Bott-periodicity, the loop of SU/SO, $\Omega(SU/SO)$, is homotopy-equivalent to BO which is the classifying space of the infinite orthogonal group. Hence we have a map

$$h: \Omega(SU(n)/SO(n)) \to BO,$$

which is induced by looping the natural inclusion map. Furthermore by Lemma 7 in [6] the above natural inclusion map is $(n-2)$-equivalence. This suggests us that we can compute the Brown-Peterson homology of $\Omega(SU(n)/SO(n))$, $BP_*\Omega(SU(n)/SO(n))$, completely by knowing BP_*BO.

In [5] the first author gave a complete answer of the Brown-Peterson homology of the classifying space BO, BP_*BO. He computed BP_*BO by using the 2-primary Adams spectral sequence. With the same techniques, we can also use the Adams spectral sequence to compute $BP_*\Omega(SU(n)/SO(n))$.

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The paper is organized as follows: In § 1 we recall the Brown-Peterson homology of the classifying space BO [5] and state the main results of this paper. In § 2 we compute the Adams E_2-term for $BP_*\Omega(SU(n)/SO(n))$. In § 3 we prove the Adams spectral sequence for $BP_*\Omega(SU(n)/SO(n))$ collapses from E_2-term, solve the group extension problem of this spectral sequence, and prove the main theorem.

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1 Statement of results

Throughout the paper homology will always have $\mathbb{Z}/2$-coefficients, and for any homology theory $h_*(\ )$, we will denote by $\tilde{h}_*(\ )$ the reduced homology.

Recall that $H_*(BO) = \mathbb{Z}/2[b_1, b_2, b_3, \ldots]$, where $|b_i| = i$, and the generators b_i come from the usual inclusion map $g: RP^\infty \to BO$. Furthermore, there is a well-known inclusion map (generating complex)

$$\bar{g}: RP^{n-1} \to \Omega(SU(n)/SO(n))$$

such that

$$H_*(\Omega(SU(n)/SO(n))) = \mathbb{Z}/2[b_1, b_2, \ldots, b_{n-1}],$$

where $|b_i| = i$, b_i comes from RP^i, $1 \le i \le n-1$, and

$$h_*: H_*(\Omega(SU(n)/SO(n))) \to H_*(BO)$$

is injective.

Let BP be the 2-primary Brown-Peterson spectrum. The basic references of Brown-Peterson homology are [2] and [4]. We now recall some results of the Brown-Peterson theory. The coefficient ring is

$$BP_* \cong \mathbb{Z}_{(2)}[V_1, V_2, V_3, \ldots], \quad |V_i| = 2(2^i - 1).$$

Since BP is a ring spectrum with complex orientation, we have

$$BP^*CP \cong BP^*[[X]],$$

where $X \in BP^2CP^\infty$ is the first Conner-Floyd class.

BP_*CP^∞ is a free BP_*-module on the generator β_i which is dual to X^i $(i \ge 0)$.

The induced map, 2^*, from the fibration,

$$RP^\infty \longrightarrow CP^\infty \xrightarrow{\times 2} CP^\infty,$$

defines $2^*(X) \equiv [2](X) \equiv \sum_{i=0}^\infty a_i X^{i+1}$, which is called 2-series and $a_i \in BP_{2i}$, $a_0 = 2$.

The reduced Brown-Peterson homology of RP^∞, $\widetilde{BP}_*RP^\infty$, is generated by

$$z_j \in \widetilde{BP}_{2j-1}RP^\infty \quad j > 0,$$

subject to the relations

$$\sum_{k=0}^{j-1} a_k z_{j-k} = 0.$$

We still use β_{2i} and z_j as the images of β_{2i} and z_j under the maps $f_*: BP_*CP^\infty \to BP_*BO$ and $g_*: BP_*RP^\infty \to BP_*BO$ respectively, where $f: CP^\infty \to BO$ and $g: RP^\infty \to BO$ are the virtual Hopf bundles of degree 0 respectively. Then we recall the result of [5]:

There is an BP_*-algebras isomorphism

$$BP_*BO \cong BP_*[\beta_{2i}, z_j]/J,$$

where $z_0 = 1$, $\deg z_j = 2j-1$ for $j \geq 1$, $\deg \beta_{2i} = 4i$ for $i \geq 1$, and J is the ideal generated by

$$\sum_{i=0}^{j-1} a_i z_{j-i} \quad \text{for } j \geq 1.$$

We now state the main results of this paper.

Theorem 1.1. *Under the loop map*

$$h: \Omega(SU(n)/SO(n)) \to BO,$$

$BP_*\Omega(SU(n)/SO(n))$ is embedded in BP_*BO, that is,

(1) *for $n = 2m$,*

$$BP_*\Omega(SU(n)/SO(n)) \cong BP_*[\beta_{2i}, z_j]/J, \quad 1 \leq i \leq m-1, 1 \leq j \leq m,$$

where $z_0 = 1$, $|z_j| = 2j-1$, $|\beta_{2i}| = 4i$, and J is the ideal generated by

$$\sum_{k=0}^{j-1} a_k z_{j-k} = 0, \quad 1 \leq j \leq m-1,$$

and

(2) *for $n = 2m+1$,*

$$BP_*\Omega(SU(n)/SO(n)) \cong BP_*[\beta_{2i}, z_j]/J, \quad 1 \leq i \leq m, 1 \leq j \leq m,$$

where $z_0 = 1$, $|z_j| = 2j-1$, $|\beta_{2i}| = 4i$, and J is the ideal generated by

$$\sum_{k=0}^{j} a_k z_{j-k} = 0, \quad 1 \leq j \leq m.$$

Remark. Since $H_*(\Omega(SU(n)/SO(n))) = \mathbb{Z}/2[b_1, b_2, \ldots, b_{n-1}]$, the top generator b_{n-1} induces a generator z_j if n is even or β_{2i} if n is odd in $BP_{n-1}\Omega(SU(n)/SO(n))$.

The idea to prove the main theorem is to use the Adams spectral sequence. We rely on the same techniques as the first author's work of BP_*BO in [5] to compute the Adams E_2-term for $BP_*\Omega(SU(n)/SO(n))$. Also with the aid of the loop map

$$h: \Omega(SU(n)/SO(n)) \to BO,$$

we could prove this Adams spectral sequence collapses from E_2 term and solve the group extension problem in this Adams spectral sequence.

2 The Adams E_2-term for $BP_*\Omega(SU(n)/SO(n))$

Let A_* be the mod 2 dual Steenrod algebra, that is $A_* = \mathbb{Z}/2[\xi_1, \xi_2, \xi_3, \ldots]$, where ξ_i is the Milnor's generator and $|\xi_i| = 2^i - 1$. The coproduct is given by

$$\Delta(\xi_i) = \sum_{k=0}^{i} \xi_{i-k}^{2^k} \otimes \xi_k .$$

Recall the 2-primary Adams spectral sequence,

$$E_2^{*,*} = \mathrm{Ext}_{A_*}^{*,*}(\mathbb{Z}/2, H_*(X)) \Rightarrow \pi_*(X) \otimes \mathbb{Z}_{(2)},$$

where $\mathbb{Z}_{(2)}$ is the integers localized at prime 2. By a well-known change-of-ring isomorphism [1], we can replace

$$\mathrm{Ext}_{A_*}^{*,*}(\mathbb{Z}/2, H_*(BP \wedge X)) \quad \text{with} \quad \mathrm{Ext}_{E}^{*,*}(\mathbb{Z}/2, H_*(X)),$$

where E is the exterior algebra of the mod 2 dual Steenrod algebra. Thus the desired Adams spectral sequence which we will use is

$$E_2^{*,*} = \mathrm{Ext}_{E}^{*,*}(\mathbb{Z}/2, H_*(X)) \Rightarrow BP_*X.$$

Recall that $|\xi_i| = 2^i - 1$ and that

$$\mathrm{Ext}_{E}^{*,*}(\mathbb{Z}/2, \mathbb{Z}/2) = \mathbb{Z}/2[w_1, w_2, w_3, \ldots],$$

where $\mathrm{bideg}(w_i) = (1, |\xi_i|)$. We will denote this ring by R.

Recall that $H_*(\Omega(SU(n)/SO(n))) = \mathbb{Z}/2[b_1, b_2, \ldots, b_{n-1}]$, where the generators b_i come from the generating complex $\bar{g}: RP^{n-1} \to \Omega(SU(n)/SO(n))$. Hence by the results of Switzer [3], the comodule structure of $H_*(\Omega(SU(n)/SO(n)))$ over the mod 2 dual Steenrod algebra is

$$\Delta(b_i) = \sum_{k=1}^{i} (\xi^k)_{i-k} \otimes b_k, \quad 1 \le i \le n-1,$$

where $\xi = 1 + \xi_1 + \xi_2 + \xi_3 + \cdots$.

In order to compute the Adams E_2-term for $BP_*\Omega(SU(n)/SO(n))$ we must understand the comodule structure of $H_*(\Omega(SU(n)/SO(n)))$ over the exterior algebra in advance. We have the exact same lemma as Lemma 2.1 of the first author's paper in [5].

Lemma 2.1. *With the notation as above, the comodule structure of $H_*(\Omega(SU(n)/SO(n)))$ over the exterior algebra E is*

$$\Delta(b_{2j-1}) = 1 \otimes b_{2j-1}, \quad 1 \le 2j-1 \le n-1,$$
$$\Delta(b_i^2) = 1 \otimes b_i^2, \quad 1 \le i \le n-1,$$
and
$$\Delta(b_{2k}) = 1 \otimes b_{2k} + \sum_{2 \le 2^l \le 2k} \xi_l \otimes b_{2k-2^l+1}, \quad 1 \le 2k \le n-1.$$

Theorem 2.2. *In the Adams spectral sequence for $BP_*\Omega(SU(n)/SO(n))$, the Adams E_2-term is*

(1) *for $n = 2m$*

$$\bar{E}_2^{*,*} \cong R \otimes \mathbb{Z}/2[\bar{\alpha}_{2i}^2, \bar{\alpha}_{2j-1}]/\bar{J}, \quad 1 \le i \le m-1, 1 \le j \le m,$$

where $\bar{\alpha}_{2i}^2$ and $\bar{\alpha}_{2j-1}$ are represented by b_{2i}^2 and b_{2j-1} in the cobar complex respectively, and $\bar{J}$ is the ideal generated by

$$\bar{\varrho}_{2j} = \sum_{2 \le 2^k \le 2j} w_k \otimes \bar{\alpha}_{2j-2^k+1}, \quad 1 \le j \le m-1.$$

(2) *for $n = 2m+1$*

$$\bar{E}_2^{*,*} \cong R \otimes \mathbb{Z}/2[\bar{\alpha}_{2i}^2, \bar{\alpha}_{2j-1}]/\bar{J}, \quad 1 \le i \le m-1, 1 \le j \le m,$$

where $\bar{\alpha}_{2i}^2$ and $\bar{\alpha}_{2j-1}$ are represented by b_{2i}^2 and b_{2j-1} in the cobar complex respectively, and $\bar{J}$ is the ideal generated by

$$\bar{\varrho}_{2j} = \sum_{2 \le 2^k \le 2j} w_k \otimes \bar{\alpha}_{2j-2^k+1}, \quad 1 \le j \le m.$$

We will only prove this theorem for the case $n = 2m$, and the proof for the case $n = 2m+1$ is similar.

To prove Theorem 2.2, we will rely on the same techniques as the paper of BP_*BO. The idea is to filter the cobar complex and to compute the desired Ext group by using the associated spectral sequence.

Let C be the cobar complex, that is, $C^k = \otimes^k \bar{E} \otimes H_*(\Omega(SU(2m)/SO(2m)))$, where $\bar{E}$ is the argumentation ideal of the exterior algebra E, and $\otimes^k \bar{E}$ is the k-fold tensor product of $\bar{E}$ $(k \ge 0)$.

Let

$$D = \mathbb{Z}/2[b_1, b_3, b_5, \ldots, b_{2m-1}]$$

be the subalgebra of $H_*(\Omega(SU(2m)/SO(2m)))$.

Now we want to define a decreasing multiplicative filtration $\{F^i\}$ on C. We do this by setting

$$F^1 C = 0, \quad \text{and} \quad F^0 C^k = \bigotimes^k \bar{E} \otimes D$$

and by defining the filtration degree of each b_{2i} $(1 \leq i \leq m-1)$ to be -1. Then by using Lemma 2.1, one can easily check the following

(1) $d(F^{-p}C^k) \subseteq F^{-p}C^{k+1}$, d is the differential in C, and

(2) $F^{-p}C^k \otimes F^{-q}C^l \to F^{-p-q}C^{k+l}$, $C^k \otimes C^l \to C^{k+l}$ is the external cup product.

So we have a spectral sequence of algebras

$$E_1^{-p,q} = H^{-p+q}(F^{-p}C/F^{-p+1}C) \Rightarrow H^{-p+q}(C).$$

where $F^{-p}C/F^{-p+1}C$ is the quotient complex, and the d_1 differential is the composite

$$E_1^{-p,q} = H^{-p+q}(F^{-p}C/F^{-p+1}C) \xrightarrow{\partial} H^{-p+q+1}(F^{-p+1}C)$$
$$\to H^{-p+q+1}(F^{-p+1}C/F^{-p+2}C) = E_1^{-p+1,q}.$$

Since in the above filtration we filter away the coaction of E,

$$E_1^{-p,*} = F^{-p}C^*/F^{-p+1}C^*,$$

that is,

$$E_1^{*,*} = R \otimes H_*(\Omega(SU(2m)/SO(2m))).$$

The proof of Theorem 2.2 now follows from the next result.

Theorem 2.3. *The filtration spectral sequence indicated above collapses from E_2-term and*

$$E_2^{*,*} \cong R \otimes \mathbb{Z}/2[b_{2i}^2, b_{2j-1}]/\bar{J}, \quad 1 \leq i \leq m-1, 1 \leq j \leq m,$$

where the ideal $\bar{J}$ is generated by

$$\bar{\varrho}_{2j} = \sum_{2 \leq 2^k \leq 2j} w_k \otimes b_{2j-2^k+1}, \quad 1 \leq j \leq m-1.$$

Lemma 2.4. *Let $\bar{R} = R \otimes D \otimes \mathbb{Z}/2[b_2^2, b_4^2, \ldots, b_{2m-2}^2]$ which is a subalgebra of $R \otimes H_*(\Omega(SU(2m)/SO(2m)))$. Let*

$$\bar{\varrho}_{2j} = \sum_{2 \le 2^k \le 2j} w_k \otimes b_{2j-2^k+1}, \quad 1 \le j \le m-1,$$

and for $2 \le s \le m$, *let* $\bar{J}_{(s-1)}$ *be the ideal of* $\bar{R}$ *generated by* $\bar{\varrho}_{2j}$, $1 \le j \le s-1$. *Then* $\bar{\varrho}_{2(s+t)}$ *is not a zero divisor in* $\bar{R}/\bar{J}_{(s-1)}$ *for* $t \ge 0$, $s+t \le m-1$ *and* $\bar{\varrho}_{2(s+t)}$ *is not in* $\bar{J}_{(s-1)}$ *for* $t \ge 0$, $s+t \le m-1$, *that is, if* $l\bar{\varrho}_{2(s+t)}$ *is in* $\bar{J}_{(s-1)}$, $l \in \bar{R}$, *then* $l \in \bar{J}_{(s-1)}$ *for* $t \ge 0$, $s+t \le m-1$.

Proof. This lemma is exact the same as Lemma 2.4 of [5] except some restrictions on the degree.

Proof of Theorem 2.3. Let

$$P_0 = R \otimes D \otimes \mathbb{Z}/2[b_2^2, b_4^2, \ldots, b_{2m-2}^2]$$

and

$$P_j = R \otimes D \otimes \mathbb{Z}/2[b_2, b_4, \ldots, b_{2j}, b_{2j+2}^2, \ldots, b_{2m-2}^2], \quad 1 \le j \le m-1$$

which are the subcomplexes of E_1-term. There are short exact sequences

$$0 \to P_j \to P_{j+1} \to P_{j+1}/P_j \to 0, \quad 1 \le j \le m-2,$$

where P_{j+1}/P_j is the quotient complex.

We will prove by induction on j that

$$H_*(P_j) = R \otimes D \otimes \mathbb{Z}/2[b_2^2, b_4^2, \ldots, b_{2m-2}^2]/\bar{J}_j, \quad 1 \le j \le m-1.$$

From Lemma 2.1, it is obvious that $H_*(P_0) = P_0$. So we can start the induction.

The short exact sequence indicated above induces a long exact sequence

$$\to H_*(P_j) \to H_*(P_{j+1}) \to H_*(P_{j+1}/P_j) \xrightarrow{\partial} H_{*-1}(P_j) \to.$$

Claim: The boundary homomorphism is multiplication by $\bar{\varrho}_{2(j+1)}$.

Note that the typical element of P_{j+1}/P_j is $x \otimes b_{2j+2}$, where $x \in P_j$. Suppose $y \in H_*(P_{j+1}/P_j)$ is represented by $x \otimes b_{2j+2}$ in the quotient complex P_{k+1}/P_k. Since

$$\begin{aligned} d_1(b_{2j+2}) &= 1 \otimes b_{2j+2} + \Delta(b_{2j+2}) \\ &= 1 \otimes b_{2j+2} + [1 \otimes b_{2j+2} + \sum_{2 \le 2^k \le 2j+2} \xi_k \otimes b_{2j-2^k+3}] = \bar{\varrho}_{2(j+1)}, \end{aligned}$$

we have

$$d_1(x \otimes b_{2j+2}) = d_1(x) \otimes b_{2j+2} + x \otimes \bar{\varrho}_{2(j+1)}.$$

But $x \otimes b_{2j+2}$ must be a cycle in the quotient complex, so $d_1(x \otimes b_{2j+2}) = 0$ in P_{j+1}/P_j, that is, $d_1(x \otimes b_{2j+2}) \in P_j$. However $b_{2j+2} \notin P_j$, so $d_1(x) = 0$, that is, $d_1(y) = x \otimes \bar{\varrho}_{2(j+1)}$.

Now by Lemma 2.4, ∂ is injective. Hence the long exact sequence

$$\to H_*(P_j) \to H_*(P_{j+1}) \to H_*(P_{j+1}/P_j) \xrightarrow{\partial} H_{*-1}(P_j) \to$$

implies that

$$H_*(P_j) \to H_*(P_{j+1})$$

is surjective. Then by the first isomorphism theorem,

$$H_*(P_{j+1}) = H_*(P_j)/Im\partial = H_*(P_j)/\langle\bar{\varrho}_{2(j+1)}\rangle,$$

where $\langle\bar{\varrho}_{2(j+1)}\rangle$ is the ideal generated by $\bar{\varrho}_{2(j+1)}$. This completes the inducitve step.

Since the filtration spectral sequence is a spectral sequence of algebras, to prove the filtration spectral sequence collapses from E_2-term, we only have to prove that b_{2i}^2 and b_{2j-1} are permanent cycles in the E_2-term of this filtration spectral sequence. Furthermore since this filtration spectral sequence converges to the Adams E_2-term for $BP_*\Omega(SU(2m)/SO(2m))$ and by Lemma 2.1 we do know that $\bar{\alpha}_{2i}^2$ and $\bar{\alpha}_{2j-1\,2}$ are in the Adams E_2-term which are detected by b_{2i}^2 and b_{2j-1}. So b_{2j}^2 and b_{2j-1} are permanent cycles in this filtration spectral sequence. This completes the proof.

3 The group extension problem in the Adams spectral sequence for $BP_*\Omega(SU(n)/SO(n))$

Recall Theorem 2.2 and Lemma 3.3 of [5].

The Adams spectral sequence for BP_*BO collapses from E_2-term and

$$E_2^{*,*} = E_\infty^{*,*} = R \otimes \mathbb{Z}/2[\alpha_{2i}^2, \alpha_{2i-1}]/\bar{J},$$

where α_{2i}^2 and α_{2i-1} are represented by b_{2i}^2 and b_{2i-1} respectively, and $\bar{J}$ is the ideal generated by

$$\bar{\varrho}_{2j} = \sum_{2\le 2^k\le 2j} \omega_k \otimes \alpha_{2j-2^k+1}\,.$$

Theorem 3.1. *The Adams spectral sequence for $BP_*\Omega(SU(n)/SO(n))$ collapses from E_2-term.*

Proof. The loop map

$$h\colon \Omega(SU(n)/SO(n)) \to \Omega(SU/SO) \simeq BO$$

induces a homomorphism of the Adams spectral sequences, it is clear that, at the E_2-level, $h_*(\bar{\alpha}_{2i}^2) = \alpha_{2i}^2$ and $h_*(\bar{\alpha}_{2j-1}) = \alpha_{2j-1}$. It follows that h_* is injective at the

E_2-level. Together with the fact that the Adams spectral sequence for BP_*BO collapses, this forces the Adams spectral sequence for $BP_*\Omega(SU(n)/SO(n))$ to collapse. This completes the proof.

Let L be a commutative ring identity, and M, N be any L-modules. Suppose M, N have decreasing filtrations respectively, that is,

$$M = M^0 \supseteq M^1 \supseteq M^2 \supseteq \dots$$

and $$N = N^0 \supseteq N^1 \supseteq N^2 \supseteq \dots .$$

Let $E^0(M)$ denote the associated graded module $\oplus_{i=0}^{\infty} M^i/M^{i+1}$.

Lemma 3.5. *Assume that $\bigcap_{i=0}^{\infty} M^i = 0$. Then if $\phi: M \to N$ is a filtered homomorphism with $E^0(\phi)$ injective, ϕ is injective.*

Proof. This is just Lemma 3.4 of [5]. This completes the proof.

Now since $\Omega((SU(n)/SO(n))$ is an H-space, $BP_*\Omega(SU(n)/SO(n))$ is an BP_*-algebra. However $\Omega(SU(n)/SO(n))$ is not a commutative H-space, for example, when $n = 2$, $SU(2)/SO(2) \cong S^3/S^1 \cong S^2$, so we don't even know whether $BP_*\Omega(SU(n)/SO(n))$ is a commutative ring or not. Now since BO is a commutative H-space, BP_*BO is a commutative BP_*-algebra. The following result implies that $BP_*\Omega(SU(n)/SO(n))$ is a commutative BP_*-algebra.

Corollary 3.6. *The induced homomorphism*

$$h_*: BP_*\Omega(SU(n)/SO(n)) \to BP_*BO$$

is injective.

Proof. From the proof Theorem 3.1, we know that $E^0(h_*)$ is injective. By Lemma 3.5, this corollary follows immediately.

Proof of Theorem 1.2. By Corollary 3.6, the induced homomorphism

$$h_*: BP_*\Omega(SU(2m)/SO(2m)) \to BP_*BO$$

is injective. So $BP_*\Omega(SU(2m)/SO(2m))$ is embedded in $BP_*[\beta_{2i}, z_j]/J$, $1 \leq i \leq m-1$, $1 \leq j \leq m$. Thus

$$BP_*\Omega(SU(2m)/SO(2m)) \cong BP_*[\beta_{2i}, z_j]/J,$$

where $1 \leq i \leq m-1$, $1 \leq j \leq m$. $1 \leq i \leq m-1$. This completes the proof for $n = 2m$.

The same argument holds for $n = 2m + 1$. This completes the proof.

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