Sophie's Diary

by Dora Musielak

BLOOMINGTON, INDIANA, AUTHORHOUSE, 2008. SOFTCOVER, 244 PP., US $11.50, ISBN: 1-4184-0812-3

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Can a fictional teenage diary of the mathematician Sophie Germain have dramatic and captivating appeal to audiences ranging from curious teenagers to professional mathematicians? The answer lies in the delicate balance between what we do and don't know about her real life, along with the extraordinary historical and mathematical circumstances that coalesced with her stranger-than-fiction initiative, perseverance, and mathematical talent, to make her the first woman we know to achieve important original mathematical research.

In order to understand the full stage setting, let us first review what we do know about Sophie Germain, including recent new discoveries about her work, and about Fermat's Last Theorem, that make her mathematical and personal story both compelling and tantalizing. What we now know leaves both abundant and timely opportunity for the creation of the book under review, a fictional diary from Sophie Germain at age 13 to 17, along with a short nonfiction appendix by the author, Dora Musielak.

The real Sophie Germain was born on April 1, 1776, and she lived with her parents and sisters in the center of Paris throughout the upheavals of the French Revolution. Even if kept largely indoors, she must as a teenager have heard, and perhaps seen, some of its most dramatic and violent events. Moreover, her father, Ambroise-François Germain, a silk merchant, was an elected member of the third estate to the Constituent Assembly convened in 1789, when the fictional diary begins [1]. He thus brought home daily intimate knowledge of events in the streets, the courts, etc.; how this was actually shared, feared, and coped with by the real Sophie Germain and her family we do not know.

Much of what we know of Germain's life comes from the biographical obituary [5] published by her friend and fellow mathematician Guglielmo Libri shortly after her death in 1831. He wrote that at age thirteen, Sophie Germain, partly as sustained diversion from her fears of the Revolution beginning outside her door, studied first Montucla's *Histoire des mathématiques*, where she read of the death of Archimedes by the sword of a Roman soldier during the fall of Syracuse, because he could not be distracted from his mathematical meditations. It seems that Sophie herself followed Archimedes, becoming utterly absorbed in learning mathematics, studying without any teacher from a then common mathematical work by Étienne Bezout that she found in her father's library.

Her family at first endeavored to thwart her in a taste so unusual and socially unacceptable for her age and sex. According to Libri, Germain rose at night to work by the glimmer of a lamp, wrapped in covers, in cold that often froze the ink in its well, even after her family had removed the fire, clothes, and candles from her room to force her back to bed. It is thus that she gave evidence of a passion they thereafter had the wisdom not to oppose. Libri writes that one often heard of the happiness with which Germain rejoiced when, after long effort, she could persuade herself that she understood the language of analysis in Bezout. Libri continues that after Bezout, Germain studied Cousin's differential calculus, and was absorbed in it during the Reign of Terror (1793–1794). Dora Musielak's diary ends on April 1, 1793, Germain's seventeenth birthday. This is a perfect ending point, since it is from roughly 1794 onwards that we have some records of Germain interacting with the public world. And it was then, Libri explains, that Germain did something so opportunistic, so rashly remarkable, so far-reaching in its consequences, that it would lack believability if it were mere fiction.

Germain, then eighteen years old, first somehow obtained the lesson books of various professors at the newly founded École Polytechnique. She particularly focused on those of Joseph-Louis Lagrange on analysis. The École, a direct outgrowth of the French Revolution, did not admit women, so Germain had no access to this splendid new institution and its faculty. However, the École did have the novel feature, heralding a modern university, that its professors were both teachers and active researchers. Indeed, its professors included some of the best scientists and mathematicians in the world. Libri writes that professors had the custom, at the end of their lecture courses, of inviting their students to present them with written observations. Sophie Germain, assuming the name of an actual student at the École Polytechnique, one Antoine-August LeBlanc, submitted her observations to Lagrange, who praised them, and learning the true name of the imposter, actually went to her to attest his astonishment in the most flattering terms.

Can we even imagine such events occurring today in fact rather than fiction? Perhaps the most astounding aspect is that Germain appears to have educated herself to at least the undergraduate level, capable of submitting written work to Lagrange, one of the foremost researchers in the world, work that was sufficiently notable to make him seek out the author. Unlike other female mathematicians before her, such as Hypatia, Maria Agnesi, and Émilie du Châtelet, who had either professional mentors or formal education, Sophie Germain appears to have climbed to university level unaided and entirely on her own initiative.

Germain's appearance on the Parisian mathematical scene, Libri continues, drew other scholars into conversation with her, and she became a passionate student of number theory with the appearance of Adrien-Marie Legendre's *Théorie des Nombres* in 1798. Both Lagrange and Legendre became important personal mentors to Germain, even though she could never attend formal courses of study. After Carl Friedrich Gauss's *Disquisitiones Artithmeticae* appeared in 1801, Germain took the additional audacious step in 1804 of writing to him, again as LeBlanc (who in the meantime had died), enclosing some research of her own on number theory, particularly on Fermat's Last Theorem. Gauss entered into serious mathematical correspondence with "Monsieur LeBlanc", whom he considered to have "completely mastered" his *Disquisitiones* [4]. In 1807 the true identity of LeBlanc was revealed to Gauss when Germain intervened with a French general to ensure Gauss's personal safety in Braunschweig during Napoleon's Jena campaign. Gauss's response to this surprise metamorphosis of his correspondent was extraordinarily complimentary and encouraging to Germain as a mathematician, and quite in contrast to the attitude of many 19th century scientists and mathematicians about women's abilities.

> But how can I describe my astonishment and admiration on seeing my esteemed correspondent Monsieur LeBlanc metamorphosed into this celebrated person, yielding a copy so brilliant it is hard to believe? The taste for the abstract sciences in general and, above all, for the mysteries of numbers, is very rare: this is not surprising, since the charms of this sublime science in all their beauty reveal themselves only to those who have the courage to fathom them. But when a woman, because of her sex, our customs and prejudices, encounters infinitely more obstacles than men, in familiarizing herself with their knotty problems, yet overcomes these fetters and penetrates that which is most hidden, she doubtless has the most noble courage, extraordinary talent, and superior genius. Nothing could prove to me in a more flattering and less equivocal way that the attractions of that science, which have added so much joy to my life, are not chimerical, than the favor with which you have honored it. The scientific notes with which your letters are so richly filled have given me a thousand pleasures. I have studied them with attention and I admire the ease with which you penetrate all branches of arithmetic, and the wisdom with which you generalize and perfect [1, p. 25].

The subsequent arcs of Sophie Germain's two main mathematical research trajectories, her interactions with other researchers, and with the professional institutions that forced her, as a woman, to remain at or beyond their periphery, are complex. Germain's development of a mathematical theory explaining the vibration of elastic membranes is told by Lawrence Bucciarelli and Nancy Dworsky in their mathematical biography [1]. And Germain's efforts to prove Fermat's Last Theorem, including recent large discoveries in her manuscripts, are told by Andrea Del Centina [2] and Reinhard Laubenbacher and David Pengelley [3, 4].

In brief, the German physicist Ernst Chladni created a sensation in Paris in 1808 with his demonstrations of the intricate vibrational patterns of thin plates, and at the instigation of Napoleon, the Academy of Sciences set a special prize competition to find a mathematical explanation. Germain pursued a theory of vibrations of elastic membranes, and based on her partially correct submissions, the Academy twice extended the competition, finally awarding her the prize in 1816 while still criticizing her solution as incomplete, and did not publish her work [1]. The whole experience was definitely bittersweet for Germain.

The Academy then immediately established a new prize, for a proof establishing Pierre de Fermat's 17th century claim that for each fixed $p > 2$, there are no positive natural number solutions to the equation $x^p + y^p = z^p$. Of course this claim, known as Fermat's Last Theorem, became one of the greatest unsolved problems in mathematics until its confirmation by Andrew Wiles at the end of the 20th century.

While Sophie Germain never submitted a solution to this new Academy prize competition and never published on Fermat's Last Theorem, we have long known that she worked on it, from a single footnote in Legendre's own 1823 memoir published on the topic [3].

Once Fermat had proven his claim for exponent 4, it could be fully confirmed just by substantiating it for odd prime exponents. But when Germain began her work, this had been accomplished only for exponent 3. Legendre's own publication proved Fermat's Last Theorem for exponent 5, but he also credited Sophie Germain with the first general result applicable to arbitrary exponents, and this has come to be known as Sophie Germain's Theorem [3]. It states that for an odd prime exponent p in the Fermat equation, if there exists an auxiliary prime θ satisfying two particular congruence conditions on the p-th power residues modulo θ, then any solution to the Fermat equation would have to have one of x, y, z divisible by p^2. Legendre also credited Germain with verifying the existence of such a θ for all $p < 100$. This theorem played an important role in work on Fermat's Last Theorem over the next two centuries.

It has long been thought that this one theorem represented Sophie Germain's entire contribution to the Fermat problem, but very recent study of her surviving manuscripts and letters has demonstrated that, on the contrary, this theorem was merely a small piece of a much larger body of work. Germain pursued nothing less than an ambitious, full-fledged plan of attack on Fermat's Last Theorem in it entirety, with extensive theoretical techniques, side results, and supporting algorithms. What we have called Sophie Germain's Theorem was only a small part of her big program, a piece that could be encapsulated and applied separately as an independent theorem, as done in print by Legendre. The much larger scope of her manuscripts was lost, but has now been rediscovered and detailed in [2, 4]. The recent resolution of the Fermat problem, and the discovery of Sophie Germain's much enlarged accomplishments on the problem, create a captivatingly timely context for Dora Musielak's book.

Sophie's Diary is delightful to read. Each section leaves one anticipating the next, wondering what will happen, whether it be the fictional Sophie's next mathematical adventure, or her recounting of and views on an episode in the saga of the French Revolution unfolding outside her door. The writing style is that of a truly curious, sensitive, and articulate young person, and the blur between fact and fiction is excellent, seductively leaving one believing that the fictional Sophie's writing is the real one's life.

The scope is huge, including four years of the fictional Sophie's mathematical self-education amidst the events of the French Revolution, ranging over the 1793 riots, the storming of the Bastille, the creation of the Constituent Assembly, the assault on Versailles, the nationalization of church property, the nobility's loss of titles, the subjugation of clergy to the state, and finally the attempted escape, imprisonment, trial, and execution of King Louis XVI. But it also addresses many broader social and political issues of the day, such as Sophie's family's views on her education, and her own essentially feminist views on how the rights of women were not being addressed by the Revolution. Despite the motto of Liberté, Égalité, Fraternité, Sophie laments the exclusion of her own aspirations as a woman to attend one of the newly founded institutions of higher education, the École Polytechnique. *Sophie's Diary* contains much speculation and questioning on Sophie's part of both non-mathematical and mathematical natures. The fictional Sophie comes across as someone who constantly challenges both herself and orthodoxy, true to what we know of the real Sophie Germain.

In the mathematical realm, the fictional Sophie begins at age 13 with ancient topics such as pi and irrationality, and progresses to challenge herself with problems that teach her about algebra, complex numbers and Euler's identity, calculus, various infinite series of Euler, analysis, differential equations, Goldbach's conjecture and quadratic forms of primes in number theory, and Pascal's triangle and combinatorics. Along the way she teaches herself Latin in order to read books by Euler like *Introductio* and *Institutiones*, and Newton's *Principia*. The reader is taken on a delightful tour of much mathematics from ancient times right up through the 18th century.

Finally, as the diary concludes on Sophie's 17th birthday, she pronounces herself ready to embark on life as a mathematician, and considers how she intends to engage those at the Academy of Sciences. This final touch is a perfect segue into the life of the real Sophie Germain, who at age 18 really did succeed in obtaining serious attention from Lagrange.

The author's historical appendix focuses on Sophie Germain's biography in the context of Fermat's Last Theorem. And she writes,

> *Sophie's Diary* was inspired by Sophie Germain. I wanted to honor Germain and make her known to generations of girls (and others as well), to promote her achievements. Knowing so little about her childhood, I wanted to present a perspective as to how the teenage Sophie must have learned mathematics on her own. Writing *Sophie's Diary* became my way of bringing Sophie to life.

Dora Musielak has admirably achieved this goal.

There are some small English and typographical errors and mathematical inaccuracies in the diary, but these would easily be remedied in a new printing. Somewhat more serious are a few historical mathematical misstatements in the appendix and on the back cover, and some confusion of wording regarding the two conditions in the hypotheses of Sophie Germain's Theorem in relation to Case 1 and Case 2 of Fermat's Last Theorem. These problems, while disappointing, can also easily be corrected.

Altogether *Sophie's Diary* is a charming, captivating book to read. It should delight mathematicians, and inspire young people, especially young women, about mathematics.

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