Fix an algebraically closed field $k$. A (one-parameter, commutative) formal group law over $k$ is an element $f = f[[X, Y]]$ such that:

1. $f(X, Y) = f(Y, X) + X + Y +$ higher order terms,
2. $f(f(X, Y), Z) = f(X, f(Y, Z)).$

Let $A$ denote the set of such formal group laws. By a theorem of Landweber, there exists a ring $L$ carrying a universal formal group law, consequently $A$ can be identified with the set of ring homomorphisms $f: L \rightarrow k$. This may be used to enrich the structure of the set $A$ to a Zariski topology, and a sheaf of rings making it into a ringed space; in fact with this data $A$ would be an algebraic space [7] in the sense of Serre, except that it is not noetherian.


Key words and phrases. Landweber-Novikov algebra, Steenrod algebra, formal group, algebraic group.

Now let $\Gamma$ denote the group of formal power series $f \in k[[T]]$ of the form $f(T) = f(T) +$ higher order terms, $n \neq 0$, the group operation being composition. This is a (nonnoetherian) algebraic group, which acts on $A$ by changing coordinates. If $f \in \Gamma$, $f = A$, then we define $f(X, Y) = f^{-1}(f(X), f(Y))$. It is not hard to see that $(f(T), f^{-1})$ defines a morphism $\Gamma \times A \rightarrow A$ of (pro-)algebraic spaces of Serre. In this note we discuss this group action when the characteristic of $k$ is positive, (a.a. that $0$ is not being involved). In a succeeding note we will apply these results to the study of unitary cobordism, via Quillen's theorem, which identifies the ring $\Lambda$ with the unitary cobordism ring of a point [6]. We remark here that $\Gamma$ is the algebraic group underlying the Landweber-Novikov algebra of operations for cobordism.

**Theorem 1.** $A$ is stratified into orbits, $A = \bigcup_{\mu \geq 0} A_{\mu}$, such that:

(a) $A_{\mu} = \bigcup_{\nu \geq 0} A_{\mu \nu}$, $A_{\mu}$ is open, and $A_{\mu}$ is closed,
(b) $A_{\mu}$ is a complete intersection of hyperplanes,
(c) $A_{\mu}$ is homogeneous under $\Gamma$, for finite $\mu$, there exists a $p$-adic $\Lambda_{\mu}$ group $G_{\mu}$ such that $(\mu, G_{\mu}) \simeq A_{\mu},$
(d) The normal bundle of $A_{\mu}$ in $A$ is given by an $(n-1)$-dimensional representation of $G_{\mu}$ over $k$.

**Remark.** The $A_{\mu}$ can be described explicitly in terms of Milnor's generators of the unitary cobordism ring; $A_{\mu}$ is the locus where $p = p_{1} = \cdots = p_{\mu} = 0$ and $p_{\mu}$ is a Milnor generator of dimension $2^{p^{\mu} - 1}$. A theorem of Landweber [6] is a corollary of the ideals $(p, p_{1}, \ldots, p_{\mu})$ in the unitary cobordism ring are the only prime ideals invariant under the Landweber-Novikov algebra.

To complete the description of the orbit structure we must identify the groups $G_{\mu}$ and the representations in (d). For this we recall that central simple division algebras over the $p$-adic numbers $Q_{p}$ are completely classified by the rank (as $Q_{p}$-vector spaces) and Brauer invariant, which lies in $Q_{Z}$.

Let $D_{\mu}$ be such a division algebra of rank $\mu$ and Brauer invariant $1_{n}$.

There is a natural valuation on $D_{\mu} = \mathbb{C}^{n} \rightarrow D_{\mu}$, the former arrow being the norm, and the latter being the valuation. Let $E_{D_{\mu}} = \{x \in D_{\mu} | \nu(x) \geq 0\}$ denote the ring of integers of $D_{\mu}$.

Now consider the twisted polynomial algebra $F_{p} = F_{p} = k$. We abbreviate by $\mathcal{A}$ the quadratic ring module the (two-sided) ideal generated by $\mathfrak{a}$. It can be shown [1, p. 52] that $F_{p} \otimes_{k} \mathcal{A}$ is isomorphic to $F_{p}$. (X), where $\nu = p^{-1} \alpha$, is defined like $\nu(k)$. Consequently $\mathcal{A}$ is a right $E_{D_{\mu}}$-module whenever $n \geq m$. 
Theorem 2. The stabilizer $G_\nu$ is isomorphic to the group of units of $E_\nu$. The representation of (16) has $M_{\nu}$ as an underlying $k$-vector space, with $G_\nu$-action given by

$$g(v) = \gamma \cdot v, \quad v \in M_{\nu}, g \in G_\nu \subset E_\nu.$$ 

Remarks. $G_\nu$ is a pro-$p$-group over $F_p$, but it can be given a $p$-adic analytic structure. As such it is a form, in the sense of Grothendieck, of $Gr(\nu, Z)$.

From the description of the normal bundle of $\Lambda_\nu$ in $A_\nu$ just given, it is possible to read on the normal representation of $\Lambda_\nu$ in $A_\nu$, $n \geq m$. It is also possible to identify the stabilizer of the infinite subgroup $\Lambda_\nu \subset G_\nu$, where $G_\nu$ is the group of units of $k(\langle G \rangle)$, considered as a profinite group over $F_p$. This is just the group underlying the reduced Steenrod algebra (at the prime $p$).

Notes on the proofs. Theorem 1(a) is just a restatement of a theorem of Lazard [4]: two formal group laws over an algebraically closed field are isomorphic iff they are of the same height. Thus $\Lambda_\nu$ is the moduli variety of formal groups of height $n$, and part (b) is proved by applying Lazard's techniques to the moduli functor of formal groups of height $n$. Part (d) is trivial.

To prove (c), we use a theorem of Grothendieck [2, 133, 134]. Let $G, X$ be respectively a group scheme and a scheme upon which $G$ acts, both noetherian over $k$, with $X$ smooth. Then $A$ is a homogeneous space of $G$ if $G(k)$ acts transitively on $X(k)$. (Here $G(k)$ and $X(k)$ are the $k$-valued points of $G, X$.)

Now the $A_\nu$ are represented by localizations of polynomial rings and are smooth, but neither they nor $\Gamma$ are noetherian. However, the above result can be extended to some semi-abelian group schemes of a certain kind. Thus we let $A_\nu(\text{deg}(r))$ denote the set of $r$-tuples of a formal group over $A_\nu$ of height $n$ ($\nu = m$). Then $A_\nu = \text{proj lim} A_\nu(\text{deg}(r))$, the maps being surjections for any $r$; it is not hard to see that $A_\nu(\text{deg}(r))$ is a smooth, noetherian scheme. Similarly, $G(k)$ is the set of invertible series in $A(k)(T^r, m)$, and acts on $A_\nu(\text{deg}(r))$, compatibly with truncations. Using these approximations systematically we prove (c).

The identification of $G_\nu$ is due to Dieudonné and Lubin; see also [7, Theorem 3, p. 72]. To identify the normal representation, we show by direct computation, following [5], that the tangent space to $A$ at $F$ is the group $Z(F, k)$ of 2-cohomology classes, while the tangent space to $A_\nu$ at $F_\nu$ is the group $B^2(F, k)$ of 2-cocycles. Thus we identify the normal bundle is given by the 2-coboundary representation $H^2(F, k)$. A basis for this group is approximately known, and one checks directly that the representation is as indicated.

BIBLIOGRAPHY

3. V. S. Giri, Modules and primitive elements in a complex coboundary.

School of Mathematics, Institute for Advanced Study, Princeton, New Jersey 08540.