Since $1_{M(q,4)}$ has order q, every element in $[M(q, 4), L_k^h(\pi)]$ must have order dividing q. This proves 2.4 if q = odd.

To prove 2.4 if $q = 2^n$ it suffices to show ∃ a premap $pf: L_k^h(\pi) \to Y$, where Y is a product of Eilenberg-Mac Lane spaces and pf induces an isomorphism (mod odd torsion) on the homotopy groups. To get pf apply the universal coefficient theorem for ordinary homology to the composition of homorphisms

$$H_*(L_k^h(\pi), \Omega_0) \otimes_Z Z_{(2)} \subset \Omega_*(L_k^h(\pi) \otimes_Z Z_{(2)} \xrightarrow{\sigma} L_{*+k}^h(\pi) \otimes Z_{(2)}.$$

Since 2.4 is true for q = odd or $q = 2^n$, it is true for all q.

Bibliography

0. D. Burghelea, R. Lashof and M. Rothenberg, *Groups of automorphisms of manifolds*; with an appendix ("The topological category") by E. Pedersen, Lecture Notes in Math., vol. 473, Springer-Verlag, Berlin and New York, 1975. MR **52** #1738.

1. W. Browder and F. Quinn, *A surgery theory for G-manifolds and stratified sets*, Manifolds-Tokyo 1973 (Proc. Internat. Conf. on Manifolds and Related Topics in Topology), Univ. Tokyo Press, Tokyo, 1975, pp. 27–36. MR **51** #11543.

2. S. E. Cappell and J. L. Shaneson, *Nonlocally flat embeddings, smoothings, and group actions*, Bull. Amer. Math. Soc. **79** (1973), 577–582. MR **47** #9635.

3. ———, *The codimension two placement problem and homology equivalent manifolds*, Ann. of Math. (2) **99** (1974), 277–348. MR **49** #3978.

4. P. E. Conner and E. E. Floyd, *Differentiable periodic maps*, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 33, Academic Press, New York; Springer-Verlag, Berlin, 1964. MR **31** #750.

5. R. C. Kirby and L. C. Siebenmann, *On the triangulation of manifolds and the Hauptvermutung*, Bull. Amer. Math. Soc. **75** (1969), 742–749. MR **39** #3500.

6. L. E. Jones, *Converse to the fixed point theorem of P. A. Smith*, Boulder, Colorado, 1970 (preprint).

7. ———, *Combinatorial symmetries of the m-disc*, Berkeley, Calif., 1971 (preprint).

8. ———, *Three characteristic classes measuring the obstruction to* PL *local unknotedness*, Bull. Amer. Math. Soc. **78** (1972), 979–980. MR **46** #6368.

9. ———, *Patch spaces: A geometric representation for Poincaré spaces*, Ann. of Math. (2) **97** (1973), 306–343. MR **47** #4269.

10. S. Maumary, *Proper surgery groups and Wall-Novikov groups*, Algebraic K-theory. III: Hermitian K-theory and geometric applications (Proc. Conf. Seattle Res. Center, Battelle Memorial Inst., 1972), Lecture Notes in Math., vol. 343, Springer, Berlin, 1973, pp. 526–539. MR **51** #14107.

11. J. W. Morgan and D. P. Sullivan, *The transversality characteristic class and linking cycles in surgery theory*, Ann. of Math. (2) **99** (1974), 463–544. MR **50** #3240.

12. F. Quinn, Thesis, Princeton Univ., Princton, N.J., 1969.

13. J. L. Shaneson, *Product formulas for $L_n(\pi)$*, Bull. Amer. Math. Soc. **76** (1970), 787–791. MR **41** #6232.

14. D. Sullivan, *Lecture notes on the characteristic variety theorem*, Princeton Univ., Princeton, N.J., 1967.

15. ———, *Geometric periodicity and the invariants of manifolds*, Manifolds—Amsterdam 1970 (Proc. Nuffic Summer School), Lecture Notes in Math., vol. 197, Springer, Berlin, pp. 44–75. MR **44** #2236.

16. L. Taylor, Thesis, Univ. of California at Berkeley, 1972.

17. C. T. C. Wall, *Surgery an compact manifolds*, Academic Press, New York, 1970.

18. R. E. Williamson, *Surgery in $M \times N$ with $\pi_1 M \neq 1$*, Bull. Amer. Math. Soc. **75** (1969), 582–585. MR **39** #4859.

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KERVAIRE'S INVARIANT FOR FRAMED MANIFOLDS*

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1. Pontryagin (see **[19]**) set up a correspondence between homotopy classes of maps $S^{n+k} \to S^n$ and bordism classes of framed k-dimensional manifolds M^k embedded in $\boldsymbol{R}^{n+k}$. When $k = 2$ and $n \geqq 2$ he constructed a map $q: H_1(M^2) \to \boldsymbol{Z}/2$ (all our homology groups will have $\boldsymbol{Z}/2$ coefficients) which is quadratic with respect to the intersection pairing. Such a quadratic form has an Arf invariant; see **[1]**, **[4]** or **[21]**. Arf(q) depends only on the framed bordism class of M and defines an isomorphism $\pi_{n+2}S^n \to \boldsymbol{Z}/2$. This procedure can be used in practice to show that certain maps are essential—for example this is done in **[13]**.

In **[10]**, M. Kervaire defined an Arf invariant for $(2l-2)$-connected, $(4l-2)$-dimensional closed manifolds which are almost parallelizable and smooth in the complement of a point. The manifolds $S^1 \times S^1$, $S^3 \times S^3$ and $S^7 \times S^7$ may be framed in different ways to have Kervaire invariant one or zero. The Kervaire invariant of any framed $(4l-2)$-manifold M was defined by first performing surgery to make M $(2l-2)$-connected. In **[10]**, Kervaire showed that his invariant vanished for closed, smooth 10-dimensional manifolds and constructed a manifold which was smooth in the complement of a point and had Kervaire invariant one. This gave his famous example of a nonsmoothable manifold.

W. Browder **[3]** extended the definition of the Kervaire invariant and also extended Kervaire's result by showing that a framed, smooth M^{4l-2} has Kervaire invariant zero if l is not a power of 2. His definition has since been extended and simplified by E. H. Brown **[5]**. Browder gave necessary and sufficient conditions in terms of the Adams spectral sequence for the existence of a framed $M^{2^{n+1}-2}$ with Kervaire invariant one. These conditions have been verified for $n = 4$ **[17]** and

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*These notes are based on a talk given by Elmer Rees.

$n = 5$ (see [15] for an account of other attempts). An explicit M^{30} was constructed in [7] and directly shown to have a framing of Kervaire invariant one; a sketch of its properties is given in §4 of these notes.

2. It is clearly desirable to be able to define the Kervaire invariant of a framed manifold directly, without having to do surgery first. It is also desirable to understand exactly how the invariant depends on the framing. The Browder-Brown approach enables one to define the quadratic form directly on the middle homology of a framed M^{2k} and to study the precise relationship between quadratic forms and framings.

First, consider quadratic forms q defined on a mod 2 vector space V relative to a nonsingular pairing $\langle\ ,\ \rangle$, i.e., $q(x + y) = q(x) + q(y) + \langle x, y\rangle$. Note that the existence of such a form implies $\langle x, x\rangle = 0$ for all $x \in V$. If q_1 and q_2 are two such forms then their difference $q_1 - q_2$ is linear and so by the nonsingularity of the pairing there is an element v such that $q_1(x) - q_2(x) = \langle x, v\rangle$. This shows that if there is one quadratic form on V then there is a 1-1 correspondence between the set of quadratic forms and V itself. It is easy to show that $\mathrm{Arf}(q_1) + \mathrm{Arf}(q_2) = q_1(v) = q_2(v)$.

Browder and Brown defined the Kervaire invariant of a framed manifold $M^{2k} \subset R^{2k+N}$ as follows. It is the Arf invariant of a quadratic form defined on $H^k(M) \cong H_k(M)$. Let ν be the normal bundle of M, $T(\nu)$ its Thom complex, and let t be the given framing. Then the Pontryagin-Thom construction gives a map $S^{2k+N} \to T(\nu)$ and t gives a homeomorphism $T(\nu) \to \Sigma^n(M_+)$ where M_+ denotes M together with a disjoint base point. Given $a \in H^k(M) = [M_+, K_k]$, where K_k denotes an Eilenberg-Mac Lane space $K(\boldsymbol{Z}/2, k)$, we may form the composite $S^{2k+N} \to \Sigma^N M_+ \to^{\Sigma^N a} \Sigma^N K_k$, which is an element of the group $\pi_{2k+N}(\Sigma^N K_k)$. A calculation shows that this group is $\boldsymbol{Z}/2$. So, from our framed manifold (M^{2k}, t), we have constructed a function $q_t\colon H^k(M) \to \boldsymbol{Z}/2$. One may check that q_t is quadratic with respect to the intersection pairing, and that its Arf invariant depends only on the framed bordism class of (M, t). We will denote this invariant by $K(M, t)$; it equals Kervaire's invariant when that is defined. The following theorem, due to E. H. Brown [5], shows the relationship between this quadratic form and those considered by Pontryagin and Kervaire.

THEOREM. *If the Poincaré dual of $a \in H^k(M)$ is represented by an embedded $S^k \subset M^{2k}$ then $q_t(a) = \varepsilon(a) + h(a)$ where*

$$\varepsilon(a) = \begin{cases} 0 & \text{if the normal bundle of } S^k \subset M^{2k} \text{ is trivial,} \\ 1 & \text{otherwise;} \end{cases}$$

$$h(a) = \begin{cases} 0 & \text{if } \varepsilon(a) = 1, \\ \text{the Hopf invariant of the framed embedding} & \\ \quad S^k \subset M^{2k} \subset R^{2k+N} \text{ if } \varepsilon(a) = 0. & \end{cases}$$

The quadratic form clearly depends at most on the framing restricted to the k-skeleton of M; it is important to understand precisely what it depends on. Browder, in [3], completely analysed this. Browder's work depends on the notion of a Wu-orientation. Note that a framing corresponds to a lift in the diagram

$$\begin{tikzcd} & O \arrow[d] \\ & EO \arrow[d] \\ M \arrow[ur, dashed] \arrow[r, "\nu"'] & BO \end{tikzcd}$$

The different choices of framing correspond to maps from M to the fibre, O, of this principal bundle. One wants structures, analogous to framings, which give quadratic forms. Since quadratic forms on $H^k(M)$ are in 1-1 correspondence with $H^k(M)$, one would like the structures to be in 1-1 correspondence with $H^k(M)$. Therefore one needs a principal fibration over BO with fibre K_k. Such fibrations correspond to $H^{k+1}(BO)$. The element chosen to classify this fibration is the Wu-class $v_{k+1} \in H^{k+1}(BO)$. The total space of this fibration is denoted by $BO\langle v_{k+1}\rangle$ and a lifting to $BO\langle v_{k+1}\rangle$ of the classifying map for the normal bundle of M is called a Wu-orientation of M. One good reason for the choice of v_{k+1} is that it is the only $(k + 1)$-dimensional characteristic class that is zero on all $2k$-manifolds. Hence every $2k$-manifold admits a Wu-orientation. A framing gives a Wu-orientation as is easily seen from the commutative diagram

$$\begin{tikzcd} EO \arrow[rr] \arrow[dr] & & BO\langle v_{k+1}\rangle \arrow[dl] \\ & BO & \end{tikzcd}$$

Let $\bar{\gamma}$ be the bundle over $BO\langle v_{k+1}\rangle$ obtained by pulling back the universal bundle over BO. A Wu-orientation gives a bundle map $w\colon \nu \to \bar{\gamma}$. Given $a \in H^k(M) = [M_+, K_k]$ we may form the composite $S^{2k+N} \to T(\nu) \to_\Delta M_+ \wedge T(\nu) \to_{a \wedge T(w)} K_k \wedge T(\bar{\gamma})$ where we have taken ν and $\bar{\gamma}$ to be N-dimensional, and Δ is induced by the diagonal. Hence the Wu-orientation w gives a function $Q_w\colon H^k(M) \to \pi_{2k+N}(K_k \wedge T(\bar{\gamma}))$. If the Wu-orientation arises from a framing then Q_w factors through the map $i\colon \pi_{2k+N}(K_k \wedge S^N) \to \pi_{2k+N}(K_k \wedge T(\bar{\gamma}))$ induced by the inclusion of a fibre of $\bar{\gamma}$. E. H. Brown [5] shows that the homomorphism i is injective (because $v_{k+1}(\bar{\gamma}) = 0$) and that one may choose an epimorphism $\varepsilon\colon \pi_{2k+N}(K_k \wedge T(\bar{\gamma})) \to \boldsymbol{Z}/4$ so that the composite $\varepsilon \circ i$ is injective. It is easily shown that the function $q_w = \varepsilon \circ Q_w$ is quadratic in the sense that $q_w(x + y) = q_w(x) + q_w(y) + j(x \cdot y)$ where $j\colon \boldsymbol{Z}/2 \to \boldsymbol{Z}/4$ is the usual inclusion. The choice of ε ensures that if the Wu-orientation came from a framing then this quadratic form takes values in $\boldsymbol{Z}/2 \subset \boldsymbol{Z}/4$ and equals the one previously defined.

The $\boldsymbol{Z}/4$ here is needed only to study quadratic phenomena on $2k$-manifolds whose kth Wu-class is nonzero. For in this case there is a class $x \in H^k(M)$ with $x \cdot x = 1$, and as we have already observed the existence of a $\boldsymbol{Z}/2$-valued quadratic form associated to the intersection pairing implies that $x \cdot x = 0$ for all $x \in H^k(M)$. Since our main interest is in framed manifolds we will assume our quadratic forms are $\boldsymbol{Z}/2$-valued.

Suppose now that we have two Wu-orientations, differing by $v \in H^k(M)$, giving quadratic forms q_1 and q_2. Then it can be shown that the quadratic forms also differ by v, that is $q_1(x) + q_2(x) = x \cdot v$. This shows that the relationship between

Wu-orientations and quadratic forms is indeed the simplest possible. So the quadratic form for a framed manifold only depends on the Wu-orientation arising from the framing. We now analyse how a change of framing affects the Wu-orientation, and this then explains the way the quadratic form depends on the framing.

Consider the commutative diagram of principal fibrations

$$\begin{tikzcd} O \arrow[r, "x_k"] \arrow[d] & K_k \arrow[d] \\ EO \arrow[r] \arrow[d] & BO\langle v_{k+1}\rangle \arrow[d] \\ BO \arrow[r, phantom, "="] & BO \end{tikzcd}$$

The fibration $BO\langle v_{k+1}\rangle \to BO$ is induced by v_{k+1} so x_k is the map Ωv_{k+1} obtained by applying the loop functor to v_{k+1}: $BO \to K_{k+1}$. Therefore if two framings of M differ by the map g: $M \to O$ then the induced Wu-orientations differ by g^*x_k. This leads to the change of framing formula:

THEOREM. *Let t_1 and t_2 be two framings of M^{2k} differing by g: $M \to O$. Then $q_{t_1}(x) + q_{t_2}(x) = x \cdot g^*x_k$ and $K(M, t_1) + K(M, t_2) = q_{t_1}(g^*x_k) = q_{t_2}(g^*x_k)$.*

An easy calculation shows that v_{k+1} is decomposable unless $k + 1$ is a power of 2. The functor Ω annihilates decomposables and so $x_k = 0$ unless $k + 1$ is a power of 2. Hence one has

COROLLARY. *The quadratic form of a framed manifold M^{2k} is independent of the framing unless $k + 1$ is a power of 2.*

Browder's theorem that the Kervaire invariant of a framed M^{4l-2} vanishes unless l is a power of 2 then follows easily from the following consequence of the Kahn-Priddy theorem due to Nigel Ray [20].

THEOREM. *If α is an element of the 2-primary component of $\pi_{n+N}(S^N)$ for $N > n + 1$, then there is a manifold M^n with framings t_1 and t_2 such that $[M, t_1] = 0$, and $[M, t_2] = \alpha \in \pi_{n+N}(S^N)$.*

The Kervaire invariant of a framed boundary is zero so Browder's theorem follows since we have shown that if l is not a power of 2 then the quadratic form and so certainly the Kervaire invariant of a framed M^{4l-2} is independent of the framing.

Conversely one can use this kind of approach to give sufficient conditions for the existence of framed manifolds M^{4l-2} with Kervaire invariant one when l is a power of 2. Perhaps the simplest is the following result of [7].

THEOREM. *There is a framed manifold M^{2k}, $k = 2^i - 1$, with Kervaire invariant one if and only if there is an element $\theta \in \pi_{2k}^s(SO)$ detected by Sq^{k+1}, i.e., the class $\mathrm{Sq}^{k+1}x_k$ is nonzero in the cofibre of θ.*

This theorem is also true with SO replaced by RP^∞. It can be used to yield another proof of Browder's result that there is a framed manifold in dimension $2^{i+1} - 2$ with Kervaire invariant one if and only if the element h_i^2 in the E_2-term of the mod 2 Adams spectral sequence survives to E_∞. The details are worked out in [7].

3. It is of interest to find criteria under which a manifold will have a framing with Kervaire invariant one. For brevity we will call a manifold "Arf-changeable" if it has framings with different Kervaire invariants. By Ray's result, if there is a framed M^{4l-2} with Kervaire invariant one, then there is an Arf-changeable manifold in the same dimension. Of course the change of framing formula gives a necessary and sufficient condition for a manifold to be Arf-changeable. There is also the following simple criterion for a manifold to be Arf-changeable. Unfortunately we have only been able to use it constructively in simple cases.

THEOREM [7]. *Let M^{2k} be a framed manifold and g_1, g_2: $M \to O$ be two maps such that $g_1^*x_k \cdot g_2^*x_k = 1$. Then M is Arf-changeable.*

PROOF. Define $\alpha_i = g_i^*x_k$, $i = 1, 2, 3$, where $g_3(x) = g_1(x)g_2(x)$. Then $\alpha_3 = \alpha_1 + \alpha_2$ and if q is a quadratic form coming from some framing then $q(\alpha_3) = q(\alpha_1) + q(\alpha_2) + 1$. Therefore it follows by the change of framing formula that one of g_1, g_2 or g_3 changes the Kervaire invariant.

COROLLARY. *If N_1^k and N_2^k are framed manifolds with $k = 1, 3$ or 7, then $N_1 \times N_2$ is Arf-changeable.*

PROOF. There are maps f_i: $N_i \to SO$ with $f_i^*x_k \neq 0$. The maps $g_1 = f_1 \circ \pi_1$ and $g_2 = f_2 \circ \pi_2$, where π_i: $N_1 \times N_2 \to N_i$ is the projection, have the required property.

We will now use a theorem due to Stong [22] to show that highly connected manifolds are not Arf-changeable.

THEOREM. *If M^{2k}, $k = 2^r - 1$, is a framed manifold and s-connected where $\phi(s + 1) \geqq r + 1$, then M is not Arf-changeable.*

So for example 8-connected M^{30}'s and 9-connected M^{62}'s are not Arf-changeable.

PROOF. Stong's theorem says that the map q_{s+1}: $BO\langle s + 1\rangle \to BO$ satisfies $q_{s+1}^* w_{2^r} = 0$ if $\phi(s + 1) > r$. Here $BO\langle s + 1\rangle$ is the $(s + 1)$th connected cover of BO. The theorem follows immediately.

An interesting class of stably parallelizable manifolds are the hypersurfaces, that is compact codimension one submanifolds of Euclidean space. If they are Arf-changeable then one can prove stronger connectivity results. Suppose $M^{2k} \subset S^{2k+1}$ is a hypersurface. Let A, B be the closures of the components of the complement of M in S^{2k+1}. The Mayer-Vietoris sequences for cohomology and real K-theory show that $i_A^* + i_B^*$: $H^k(A) \oplus H^k(B) \to H^k(M)$ and $i_A^* + i_B^*$: $KO^{-1}(A) \oplus KO^{-1}(B) \to KO^{-1}(M)$ are isomorphisms, where i_A: $M \to A$ and i_B: $M \to B$ are the inclusions of the boundaries. So given g: $M \to O$ write $g = g_1 + g_2$ where $g_1 = i_A \circ g_A$ and $g_2 = i_B \circ g_B$ where g_A: $A \to O$, g_B: $B \to O$. Therefore $g^*x_k = i_A^*\alpha_A + i_B^*\alpha_B$, where $\alpha_A = g_A^*x_k$ and $\alpha_B = g_B^*x_k$. The natural framing of M^{2k} coming from its embedding in S^{2k+1} extends over both A and B. Let q be the quadratic form coming from this framing; then it follows that q vanishes on both $i_A^*H^k(A)$ and $i_B^*H^k(B)$. Therefore

$$q(g^*x_k) = q(i_A^*\alpha_A + i_B^*\alpha_B) = i_B^*\alpha_B \cdot i_A^*\alpha_A = g_1^*x_k \cdot g_2^*x_k.$$

It follows that if a hypersurface is Arf-changeable then one may always use the above theorem from [7] to prove it.

PROPOSITION. *If M^{30} is a 7-connected hypersurface then it is not Arf-changeable.*

PROOF. Let $g_1, g_2 \colon M \to 0$ be maps; then from Stong **[22]** we know that if $\alpha_i = g_i^* x_{15}$ then $\alpha_i = (\mathrm{Sq}^7 + \mathrm{Sq}^4\mathrm{Sq}^2\mathrm{Sq}^1)\beta_i$ for some $\beta_i \in H^8(M)$ and further $\mathrm{Sq}^2\beta_i = 0$. The result is proved by showing $((\mathrm{Sq}^7 + \mathrm{Sq}^4\mathrm{Sq}^2\mathrm{Sq}^1)\beta_1) \cdot ((\mathrm{Sq}^7 + \mathrm{Sq}^4\mathrm{Sq}^2\mathrm{Sq}^1)\beta_2) = 0$ using the parallelizability of M and the Adem relations.

4. In **[7]** the ideas we are describing are used successfully to show that a certain explicitly constructed M^{30} is Arf-changeable. We now describe this manifold. Consider X_5 the orientable surface of genus 5; it has a smooth free action of the dihedral group D_4 (the symmetries of a square). The quotient space of this action is the non-orientable surface Y_1 of Euler characteristic -1, the connected sum of the projective plane and the torus. The action is best described by giving a homomorphism $\phi \colon \pi_1 Y_1 \to D_4$. The group $\pi_1 Y_1$ has generators A_1, A_2, B and one relation: $A_1 A_2 A_1^{-1} A_2^{-1} = B^2$. The group D_4 permutes the four vertices of the square and in this way can be regarded as a subgroup of S_4. The homomorphism ϕ is then given by $\phi(A_1) = (14)(23)$, $\phi(A_2) = (13)(24)$ and $\phi(B) = (13)$. In the following one may assume that X_5 is the D_4 covering of Y_1 associated with ϕ; the fact that X_5 has genus 5 is not necessary.

We now define M^{30} to be $X_5 \times_{D_4} (S^7)^4$ where D_4 acts as a permutation group on $(S^7)^4$. It is readily checked that M^{30} is stably parallelizable. Moreover one can construct a map $g \colon M^{30} \to SO$ as follows:

$$M^{30} \xrightarrow{1\times\omega^4} X_5 \times_{D_4} (SO(8))^4 \xrightarrow{\bar\phi\times 1} ED_4 \times_{D_4} (SO(8))^4 \xrightarrow{D} SO(32).$$

The first map is induced by a map $\omega \colon S^7 \to SO(8)$ such that $\omega^* x_7$ is nontrivial, $\bar\phi \colon X_5 \to ED_4$ is the equivariant map that covers ϕ and D is a finite version of the Dyer-Lashof map for SO and is described explicitly in **[14]**. Using the results of **[12]**, it is straightforward to calculate $\alpha = g^* x_{15}$ once one knows enough about $H^*(M)$. It turns out that if q is any quadratic form coming from a framing then $q(\alpha) \neq 0$. The change of framing formula shows that this M^{30} is Arf-changeable. Although the method outlined here only proves that this M^{30} is Arf-changeable, it is in fact possible with a little care to identify a framing on M which has Kervaire invariant one.

In the calculation of $q(\alpha)$ one can use the following lemma which may be of independent interest.

LEMMA [7]. *If q comes from a framing on M^{4l-2} then $q(\mathrm{Sq}^1 a) = a \cdot \mathrm{Sq}^2 a$ for any $a \in H^{4l-2}(M)$.*

PROOF. One considers the map $\mathrm{Sq}^1_* \colon \pi^s_{4l-2}(K_{2l-2}) \to \pi^s_{4l-2}(K_{2l-1})$ and checks that this is an epimorphism. Moreover an element θ that maps nontrivially is such that $\theta^*(\iota_{2l-2} \cdot \mathrm{Sq}^2 \iota_{2l-2}) \neq 0$.

The first-named author has recently proved several further results of this kind.

COROLLARY. *If M^{4l-2} is stably parallelizable and $a \in H^{2l-2}(M)$ is such that $\mathrm{Sq}^1 a = 0$ then $a \cdot \mathrm{Sq}^2 a = 0$.*

This corollary for l odd can be deduced from Theorem 1.2 of **[16]** and for $l = 3$ is given in **[18]**. It is of interest to note that this corollary is false in general for manifolds all of whose Stiefel-Whitney classes vanish; the manifolds CP^n for $n + 1$ a power of 2 are examples. It is also false for manifolds M^{4l} as the example $SU(3)$ shows. It might be interesting to construct examples in other dimensions.

REFERENCES

1. C. Arf, *Untersuchungen über quadratische formen in Körpen der charakteristik* 2, J. Reine Angew Math. **183** (1941), 148–167. MR **4,** 237.

2. M. F. Atiyah, *Thom complexes*, Proc. London Math. Soc. (3) **11** (1961), 291–310. MR **24** #A1727.

3. W. Browder, *The Kervaire invariant of framed manifolds and its generalisations*, Ann. of Math. (2) **90** (1969), 157–186.

4. ———, *Surgery on simply connected manifolds*, Springer-Verlag, Berlin and New York, 1972.

5. E. H. Brown, *Generalisations of the Kervaire invariant*, Ann. of Math. (2) **95** (1972), 368–383. MR **45** #2719.

6. F. Hirzebruch and K. H. Mayer, *O(n)-mannigfaltigkieten, exotische sphären und singularitäten*, Lecture Notes in Math., vol. 57, Springer-Verlag, Berlin and New York, 1968.

7. J. Jones, D. Phil. Thesis, Oxford Univ., 1976.

8. J. Jones and E. Rees, *A note on the Kervaire invariant*, Bull. London Math. Soc. **7** (1975), 279–282. MR **52** #6759.

9. D. S. Kahn and S. B. Priddy, *Applications of the transfer to stable homotopy theory*, Bull. Amer. Math. Soc. **78** (1972), 981–987. MR **46** #8220.

10. M. Kervaire, *A manifold which does not admit any differentiable structure*, Comment. Math. Helv. **34** (1966), 256–270.

11. M. Kervaire and J. Milnor, *Groups of homotopy spheres*. I, Ann. of Math. (2) **77** (1963), 504–537.

12. S. O. Kochman, *Homology of the classical groups over the Dyer-Lashof algebra*, Trans. Amer. Math. Soc. **185** (1973), 83–137. MR **48** #9719.

13. K. Y. Lam, *Some interesting examples of non-singular bilinear maps*, Topology **16** (1977), 185–189.

14. I. Madsen, *On the action of the Dyer-Lashof algebra in H_*G*, Pacific J. Math. **60** (1975), 235–277. MR **52** #9228.

15. M. E. Mahowald, *Some remarks on the Kervaire invariant problem from the homotopy point of view*, Proc. Sympos. Pure Math., vol. 22, Amer. Math. Soc., Providence, R. I., 1971, pp. 165–169.

16. M. E. Mahowald and F. P. Peterson, *Secondary cohomology operations on the Thom class*, Topology **2** (1963), 367–377. MR **28** #612.

17. M. Mahowald and M. Tangora, *Some differentials in the Adams spectral sequence*, Topology **6** (1967), 349–369.

18. S. Morita, *Smoothability of* PL *manifolds is not topologically invariant*, Manifolds-Tokyo, 1973, Univ. of Tokyo Press, Tokyo, 1975, pp. 51–56. MR **51** #6837.

19. L. Pontrjagin, *Smooth manifolds and their applications in homotopy theory*, Amer. Math. Soc. Transl. (2) **11** (1959), 1–114. MR **22** #5980.

20. N. Ray, *A geometrical observation on the Arf invariant of a framed manifold*, Bull. London Math. Soc. **4** (1972), 163–164.

21. C. P. Rourke and D. P. Sullivan, *On the Kervaire obstruction*, Ann. of Math. (2) **94** (1971), 397–413. MR **46** #4546.

22. R. E. Stong, *Determination of $H^*(BO(k, \cdots,), Z_2)$ and $H^*(BU(k, \cdots,), Z_2)$*, Trans. Amer. Math. Soc. **107** (1963), 526–544. MR **27** #1944.

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