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Equivariant Cohomology Theories

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Preface

These notes constitute the lecture notes to a series of lectures which the author gave at Berkeley in the spring of 1966.

Our central objective is to provide machinery for the study of the set [[X;Y]] of equivariant homotopy classes of equivariant maps from the G-space X to the G-space Y (with base points fixed by G). (For various reasons we restrict our attention to the case in which G is a finite group.) An important tool for this study is equivariant cohomology theory. It is immediately seen, however, that the classical equivariant cohomology theory is quite inadequate for the task.

Our first object then is to develop an "equivariant classical cohomology theory" (as opposed to "classical equivariant cohomology theory") which is readily computable and which, for example, allows the development of an equivariant obstruction theory. This is done in Chapter I and the obstruction theory is considered in Chapter II. Our cohomology theory includes the classical theory as a special case.

An approximation to [[X;Y]] is the stable object $\lim[[S^nX;S^nY]]$ which forms a group. If Y is a sphere, with a given G-action, this leads to the stable equivariant cohomotopy groups of a G-space X. These form an "equivariant generalized cohomology theory" and such theories are considered briefly in Chapter IV and related to the equivariant classical cohomology.

When X and Y are both spheres with (standard) involutions on them, the groups $\lim[[S^nX;S^nY]]$ are analogues of the stable homotopy groups of spheres and constitute the case of greatest interest to us at present. It is in fact this case which inspired the general theory expounded in these notes. Originally we intended to include a fifth chapter in these notes which would apply the general theory to this special case. However, the special case has since expanded in length and in importance to the extent that we have decided to publish our results on this topic separately. An outline of these results has appeared in our research announcement "Equivariant stable stems" in Bull. Amer. Math. Soc. 73 (1967) 269-273.

The main results in the present notes have been announced in "Equivariant cohomology theories," Bull. Amer. Math. Soc. 73 (1967) 266-268. Although we have restricted our attention, in these notes, to the case of finite groups it will be apparent that the theory goes through for cellular actions of discrete groups and this fact was incorporated in our research announcement (*loc. cit.*).

Those sections of the notes which contain relatively inessential material are marked with an asterisk. During the work on this subject the author was partially supported by the National Science Foundation grant GP-3990 and by a fellowship from the Alfred P. Sloan Foundation.

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Chapter IV. Generalized Equivariant Cohomology