MAY: EQVR STABLE HTY THEORY

Note Title 10/25/2010

Some history: Adams introduced ASS to prove Hopf invariant 1
$d_2(h_i) = h_0 h_{i-1}^2$, but did not state it in this way in HID paper.
HHR prove $d_?(h_1^2) \neq 0$ with no clue about the RHS. If θ_6 does not exist then we might be able to guess the general picture

Have written a precis of Mandell-May online at math.uchicago.edu/~may/KERVAIRE "Equivariant orthogonal spectra and S-modules". It describes only the portion of the theory relevant to HHR.

C_G = category where objects have G-actions as do mapping spaces $F(X,Y)$ (by conjugation)

$F(X,Y)^G$ = space of G-maps

This category does not have limits,

colimits, etc.

$G\mathcal{C}$ = category of G-spaces + G-maps

(our $\mathcal{C}^G$)

$\mathcal{D}_G$ = small equiv category

And we consider functors $\mathcal{D}_G \to \mathcal{C}$

$\mathcal{U}$ = universe $= \mathbb{R}^\infty \oplus \cdots$

$\mathcal{V}(\mathcal{U})$ = all subreps of $\mathcal{U}$

$\mathcal{I}_G(V, W)$ = space of linear isometric isomorphisms

= G-space

An $\mathcal{I}_G$-space is a functor

$\mathcal{L}_G \longrightarrow \mathcal{T}$ = based top. spaces.

$V \longmapsto X_V$

$\mathcal{L}_G(V,W) \xrightarrow{G\text{-maps}} \mathcal{T}_G(X_V, X_W)$

Example: $V \longmapsto S^V$ = one pt compactification of V

$S^V \wedge S^W \longrightarrow S^{V \oplus W}$ For a G-space A

$\Sigma^V A = A \wedge S^V \qquad \Omega^V A = F(S^V, A)$

An $\mathcal{L}_G$-spectrum has $X_V \wedge S^W \longrightarrow X_{V \oplus W}$

$$\underset{\Sigma^W X_V}{\overset{X_V \wedge S^W}{\parallel}}$$

This is a co-ord free notion of spectra.

Equiv Atiyah duality requires equiv desuspension.

$\mathcal{J}_G$-space is an $\mathcal{A}_G$-spectrum

$$S^{-V} = V^* = \text{functor } W \longrightarrow F(S^V, S^W)$$
$$= F_V S^0$$
$$= \text{left adjoint to evaluation at } V.$$

$$\mathcal{J}_G(V,W) = \mathcal{A}_G\text{-spectra}\,(S^{-W}, S^{-V})$$
$$\| $$
$$\mathcal{A}_G A\,(S^{-W}, S^{-V})$$

$\pi_n X = \operatorname{colim} \pi_n(\Omega^R \Sigma^R X)$

Model structure

A weak equiv of orth spectra is a map inducing iso in π_*^H for all $H \subseteq$

A spectrum with a G-action (a naive G-spectrum) only makes use of trivial reps.

Quillen model structure

Weak equiv $X^H \xrightarrow{\text{w.e.}} Y^H$ is a weak equiv $\forall H \subset G$

fibr. $X^H \longrightarrow Y^H$ is Serre fibr "

$\{ G/H_+ \wedge S^n_+ \longrightarrow G/H_+ \wedge D^{n+1}_+ \} =: I$

cofibs rel. cell

$\{G/H_+ \wedge D^n_+ \longrightarrow G/H_+ \wedge (D^n \times I)_+\} =: J$ build up general cofibs.

$FI = \{S^{-V} \wedge i \mid i \in I\}$

$FJ = \{S^{-V} \wedge j \mid j \in J\}$

HHR use different model structure
Fibrant objects are Ω-spectra

Have not discussed symmetric monoidal property.

$X_V \wedge Y_W$ is $\mathcal{L}_G \times \mathcal{L}_G$-space

$$\begin{tikzcd} \mathbb{1}_G \times \mathbb{1}_G \arrow[rr, "X \wedge Y"] \arrow[d, "\Phi"] & & G\mathcal{J} \\ \mathbb{1}_G \arrow[urr, dashed, "X \wedge Y"'] & & \end{tikzcd}$$

$$X \wedge S \wedge Y \rightrightarrows X \wedge Y \longrightarrow X \wedge_S Y \quad \text{coequalizer}$$

change of gps + change of universes

$$H \hookrightarrow G : \quad (G_+ \wedge_H Y)(V) = G_+ \wedge_H Y(i^* V)$$

etc