# Hill - MFO - 9-21-10

Note Title 9/21/2010

<u>Thm</u> If we consider a regular rep sphere $S^{k\rho} \wedge H\mathbb{Z}$ then $\pi_{-2} = 0$ for any $k$, $\rho = \rho_G$, $G = C_{2^{n+1}}$ for $n \geq 0$.

<u>Cor</u> $\pi_{-2}^G MU^{(2^n)} = \pi_{-2}^G \Sigma^{k\rho} MU^{(2^n)} = 0 \quad \forall k$

<u>Gap Thm</u> If $\bar{\Delta} : S^{m\rho} \longrightarrow MU^{(2^n)}$ then $\pi_{-2}^G \bar{\Delta}^{-1} MU^{(2^n)} = 0$. Similarly for $\pi_{-3}$ and $\pi_{-1}$.

Proof $\bar{\Delta}^{-1} MU^{(2^n)} = \varinjlim \Sigma^{-km\rho} MU^{(2^n)}$

$\pi_{-2}(\quad) = \varinjlim \pi_{-2}(\quad) = 0$ QED

We need to discuss slice differentials, so we need to name some elements.

If $V$ is a rep, $a_V$ denotes $S^0 \longrightarrow S^V$. Iff $V$ contains a trivial rep, then $a_V = 0$.

$a_{V \oplus W} = a_V a_W$. In $\pi^G_{-V} S^0$, $a_V$ has infinite order

- If $V$ is orientable rep of $G = C_{2^{n+1}}$, then $\pi_{\dim V}(S^V \wedge H\underline{\mathbb{Z}}) = \mathbb{Z}$, i.e. there is an orientation

$$S^{\dim V} \xrightarrow{u_V} S^V \wedge H\underline{\mathbb{Z}}$$

. If $V = 1 + W$, then $u_W = u_V$

$u_{V+W} = u_V u_W$. This element is not in the Hurewicz image

Example $G = C_4$, $V = 2\rho$

Chain complex $C_*(S^{2\rho})$

$$
\begin{array}{ccccccccccccc}
2 & & 3 & & 4 & & 5 & & 6 & & 7 & & 8 \\
\mathbb{Z} & \xleftarrow{\text{fold}} & \mathbb{Z}^2 & \xleftarrow{1-\gamma} & \mathbb{Z}^2 & \xleftarrow{1+\gamma} & \mathbb{Z}^4 & \xleftarrow{1-\gamma} & \mathbb{Z}^4 & \xleftarrow{T=(1+\gamma)(1+\gamma^2)} & \mathbb{Z}^4 & \xleftarrow{1-\gamma} & \mathbb{Z}^4 \\
 & & & & & & \parallel & & & & & & \\
 & & & & & & \mathbb{Z}[G] & & & & & & \\
S^2 & & S^{2+\sigma} & & S^{2+2\sigma} & & — & & S^{2+2\sigma+\lambda} & & — & & S^{2+2\sigma+2\lambda} \\
 & & & & & & & & & & & & \parallel \\
 & & & & & & & & & & & & S^{2\rho}
\end{array}
$$

Fixed points

$$
\mathbb{Z} \xleftarrow{2} \mathbb{Z} \xleftarrow{0} \mathbb{Z} \xleftarrow{4} \mathbb{Z} \xleftarrow{0} \mathbb{Z} \xleftarrow{4} \mathbb{Z} \xleftarrow{0} \mathbb{Z}
$$

name of generators

$$
a_{2\sigma+2\lambda} \quad — \quad a_{2\lambda} u_{2\sigma} \quad — \quad a_{\lambda} u_{2\sigma+\lambda} \quad — \quad u_{2\sigma+2\lambda}
$$

$$S^W \xhookrightarrow{a_V} S^{V+W}$$

$$C_*(S^W) \xrightarrow{a_V} C_*(S^{V+W})$$

## Dugger's example

$K_{\mathbb{R}}$ = Atiyah's real K-theory

$G = C_2$: Slice decomposition is

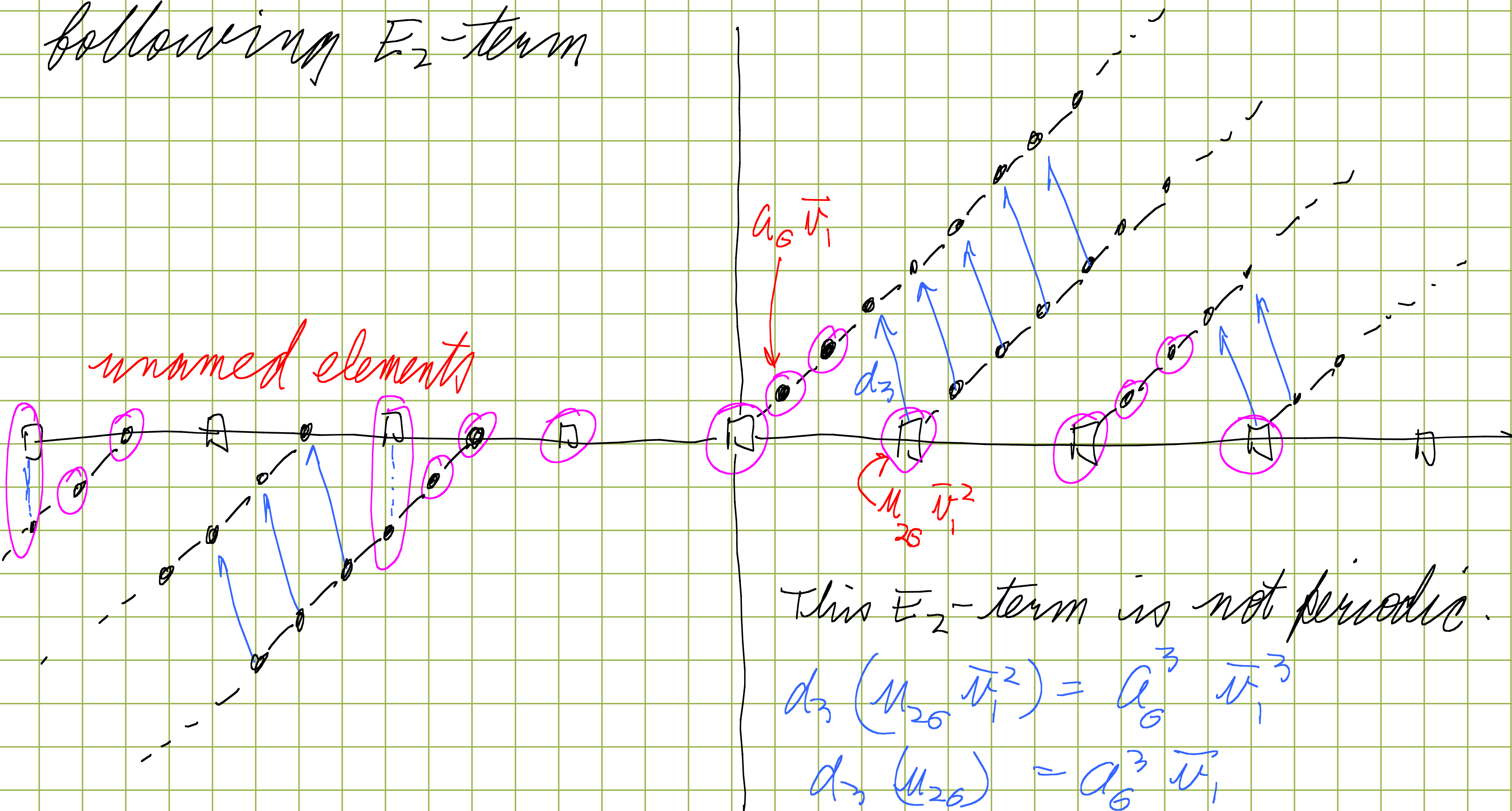
$$\bigvee_{n \in \mathbb{Z}} S^{n\rho} \wedge H\underline{\mathbb{Z}}$$

We have equivariant map $S^\rho \xrightarrow{\bar{v}_1} K_{\mathbb{R}}$

refined the Bott element $S^2 \xrightarrow{v_1} K$

We will use this to compute $\pi_* K_{\mathbb{R}}^{C_2}$

This leads to the following $E_2$-term




This $E_2$-term is not periodic.

$d_3(u_{26} \bar{v}_1^2) = a_6^3 \bar{v}_1^3$

$d_3(u_{26}) = a_6^3 \bar{v}_1$

The differential above (which for now is sleight of hand) leads to $\pi_* K_{\mathbb{R}}^{C_2}$ and we see it is the same as $\pi_* K_{\mathbb{R}}^{hC_2} = \pi_* KO$

More generally we have $S^{2i} \xrightarrow{r_i} MU^{(2^n)}$

refining to a $C_2$-map $S^{i\rho_2} \xrightarrow{\bar{r}_i} MU^{(2^n)}$

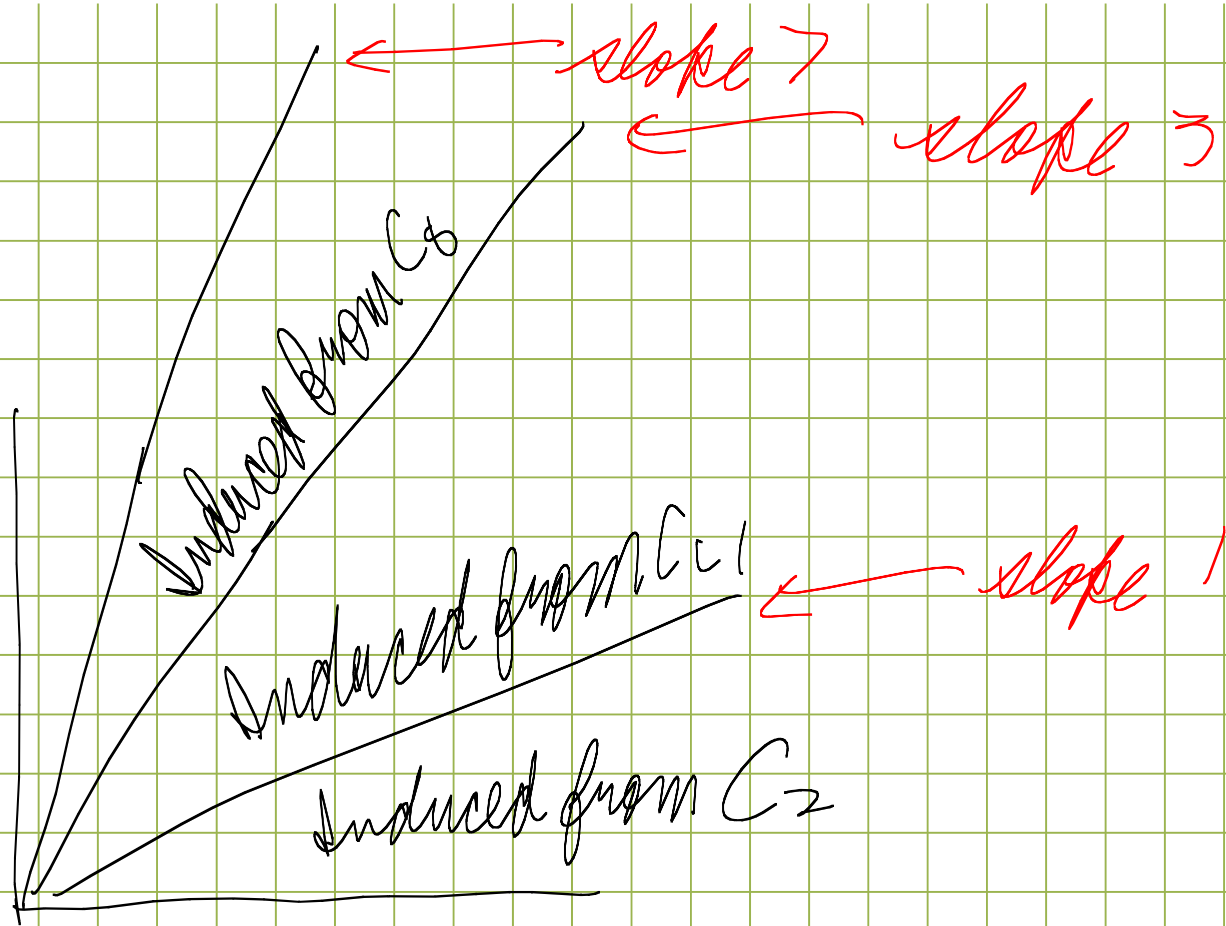
We get a $G$-map $$G_+ \wedge_{H_p} S^{\frac{|P|}{|H_p|}\rho_{H_p}} \longrightarrow MU^{(2^n)}$$

In the SS,

everything induced from $C_2$ lies below a line of slope $1$

'' $C_4$ '' $3$

'' $C_8$ $7$




$N(\bar{m}_1) : S^{\rho_G} \longrightarrow MU^{(\mathbb{Z}^n)}$

$\shortparallel$

$m_1 \cdot \gamma m_1 \cdot \gamma^2 m_1 \cdots$

# Geometric fixed points have these properties

| $Y$ | Fixed pts $(Y)^G$ | Geo fixed pts $\Phi^G(Y)$ |
|---|---|---|
| $G_+ \wedge_H (X)$ | $(\ \ )^G$ not nec trivial | $*$ |
| $\Sigma^\infty_+ X$ | $(\Sigma^\infty_+ X)^G = ?$ | $\Sigma^\infty_+ (X^G)$ |
| $X \wedge Y$ | $(X \wedge Y)^G = ?$ | $\Phi^G(X \wedge Y) = \Phi^G X \wedge \Phi^G Y$ |

$$\Phi^{C_2} MU = MO$$

$$\Phi^{G} MU^{(2^n)} = MO$$