Annals of Mathematics, **142** (1995), 425–426

Erratum
Stable K-theory and topological Hochschild homology

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Lannes and Oliver have pointed out to us that although F respects products, each $F_q(-)$ will not. Hence the use of additivity in the proof of Theorem 2.6 is not correct. However, by stabilizing in the S direction we get a statement which is equally suited for the applications:

THEOREM 2.6$'$. *For any exact category $\mathcal{C}$, the natural map by degeneracies*

$$\lim_{k\to\infty} \Omega^k F_0(S^{(k)}\mathcal{C}) \to \lim_{k\to\infty} \Omega^k F(S^{(k)}\mathcal{C}) \simeq \Omega F(S\mathcal{C})$$

is a homotopy equivalence.

Proof. More generally, we show that for all n the map

$$\sigma\colon \lim_{k\to\infty} \Omega^k F_0(S^{(k)}\mathcal{C}) \to \lim_{k\to\infty} \Omega^k F_n(S^{(k)}\mathcal{C})$$

given by degeneracies is an equivalence, which implies the result. The map is split by the face maps, sending $(\alpha_0;\alpha_1,\dots\alpha_n) \in F_n(S^{(k)}\mathcal{C})$ to $\delta(\alpha_0;\alpha_1,\dots,\alpha_n) = (\alpha_0\cdots\alpha_n) \in F_0(S^{(k)}\mathcal{C})$. We need to show that $\sigma\circ\delta \sim \mathrm{id}$.

Let X be any functor from exact categories to (simplicial) abelian groups satisfying $X(0)=0$. Regarding $S^{(k)}\mathcal{C}$ as a k multisimplicial exact category, we see that

$$X(S^{(k)}\mathcal{C}\times S^{(k)}\mathcal{D}) \to X(S^{(k)}\mathcal{C})\times X(S^{(k)}\mathcal{D})$$

is $2k$ connected since the source and target, viewed as $2k$ multisimplicial groups, agree in total degree less than $2k$. This means that under the weakened assumptions Lemma 2.2 should read "$XS^{(k)}S_2(\mathcal{C}) \to XS^{(k)}\mathcal{C}\times XS^{(k)}\mathcal{C}$ is $2k$ connected", and Lemma 2.5 should read "$d_0+d_2 \simeq d_1\colon \lim_{k\to\infty}\Omega^k XS^{(k)}S_2 \to \lim_{k\to\infty}\Omega^k XS^{(k)}$" where Ω is a model for the loops within simplicial abelian groups.

Now, letting $X=F_n$, we define two natural transformations

$$T_{\mathrm{id}}, T_\beta\colon \lim_{k\to\infty}\Omega^k F_n S^{(k)} \to \lim_{k\to\infty}\Omega^k F_n S^{(k)} S_2$$

*Originally published November 1994, **140**, No. 3.

induced by the natural transformations $t_{\mathrm{id}}, t_\beta \colon F_n \to F_n S_2$ given by sending an element $(\alpha_0; \alpha_1, \dots, \alpha_n) \in F_n\mathcal{C}$ to

$$t_{\mathrm{id}}(x) = \left\{ \begin{array}{ccccccccccc}
0 & \longleftarrow & c_p & = & c_p & = & \cdots & = & c_p & \longleftarrow & 0 \\
\downarrow & & \downarrow{\scriptstyle i_1} & & \downarrow{\scriptstyle i_1} & & \downarrow{\scriptstyle i_1} & & \downarrow & & \\
c_p & \xleftarrow{\alpha_0\pi_2} & c_p \oplus c_0 & \xleftarrow{\mathrm{id}\oplus\alpha_1} & c_p \oplus c_1 & \xleftarrow{\mathrm{id}\oplus\alpha_2} & \cdots & \xleftarrow{\mathrm{id}\oplus\alpha_{p-1}} & c_p \oplus c_{p-1} & \xleftarrow{(\mathrm{id}\oplus\alpha_p)\Delta} & c_p \\
\| & & \downarrow{\scriptstyle \pi_2} & & \downarrow{\scriptstyle \pi_2} & & \downarrow{\scriptstyle \pi_2} & & \| & & \\
c_p & \xleftarrow{\alpha_0} & c_0 & \xleftarrow{\alpha_1} & c_1 & \xleftarrow{\alpha_2} & \cdots & \xleftarrow{\alpha_{p-1}} & c_{p-1} & \xleftarrow{\alpha_p} & c_p
\end{array} \right.$$

and

$$t_\beta(x) = \left\{ \begin{array}{ccccccccccc}
c_p & \xleftarrow{\beta} & c_p & = & c_p & = & \cdots & = & c_p & = & c_p \\
\| & & \downarrow{\scriptstyle (1\oplus\beta_1)\Delta} & & \downarrow{\scriptstyle (1\oplus\beta_2)\Delta} & & & & \downarrow{\scriptstyle (1\oplus\beta_p)\Delta} & & \| \\
c_p & \xleftarrow{\alpha_0\pi_2} & c_p \oplus c_0 & \xleftarrow{\mathrm{id}\oplus\alpha_1} & c_p \oplus c_1 & \xleftarrow{\mathrm{id}\oplus\alpha_2} & \cdots & \xleftarrow{\mathrm{id}\oplus\alpha_{p-1}} & c_p \oplus c_{p-1} & \xleftarrow{(\mathrm{id}\oplus\alpha_p)\Delta} & c_p \\
\downarrow & & \downarrow{\scriptstyle \beta_1 - 1} & & \downarrow{\scriptstyle \beta_2 - 1} & & & & \downarrow{\scriptstyle \beta_p - 1} & & \downarrow \\
0 & \longleftarrow & c_0 & \xleftarrow{\alpha_1} & c_1 & \xleftarrow{\alpha_2} & \cdots & \xleftarrow{\alpha_{p-1}} & c_{p-1} & \longleftarrow & 0
\end{array} \right.$$

where i_j (resp. π_j) is the j^{th} inclusion (resp. projection), Δ is the diagonal and $\beta_k = \prod_{k \le i \le q} \alpha_i$. Note the identities

$$d_0 T_{\mathrm{id}} = \mathrm{id}, \qquad d_2 T_\beta = \sigma \circ \delta, \qquad d_2 T_{\mathrm{id}} = d_0 T_\beta = 0 \qquad \text{and} \qquad d_1 T_{\mathrm{id}} = d_1 T_\beta.$$

Hence

$$\mathrm{id} \;=\; d_0 T_{\mathrm{id}} \simeq d_1 T_{\mathrm{id}} = d_1 T_\beta \simeq d_2 T_\beta = \sigma \circ \delta. \qquad \square$$

We similarly change Definition 3.2 to

DEFINITION 3.2′. *For M an R bimodule, we let*

$$\mathrm{THH}(R;M) = \lim_{k\to\infty} \Omega^k \left| \bigoplus_{c \in \mathrm{ob}\, S^{(k)}\mathcal{C}} \mathrm{Hom}_{S^{(k)}\mathcal{C}}(c, c \otimes_R M) \right|$$

(that is: $\mathrm{THH}(R, M) = \lim_{k\to\infty} \Omega^k \, \mathrm{THH}^{(k)}$ using the notation from the proof of Theorem 3.4); the rest of the argument follows with minor changes as outlined above.

(Received April 12, 1995)