

Erratum: Stable K-Theory and Topological Hochschild Homology

Author(s): Bjørn Ian Dundas and Randy McCarthy

Source: *Annals of Mathematics*, Vol. 142, No. 2 (Sep., 1995), pp. 425-426

Published by: Mathematics Department, Princeton University

Stable URL: <https://www.jstor.org/stable/2118639>

Accessed: 19-10-2024 11:17 UTC

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# Erratum

## Stable K-theory and topological Hochschild homology

By BJØRN IAN DUNDAS AND RANDY MCCARTHY\*

Lannes and Oliver have pointed out to us that although  $F$  respects products, each  $F_q(-)$  will not. Hence the use of additivity in the proof of Theorem 2.6 is not correct. However, by stabilizing in the  $S$  direction we get a statement which is equally suited for the applications:

**THEOREM 2.6'.** *For any exact category  $\mathcal{C}$ , the natural map by degeneracies*

$$\lim_{k \rightarrow \infty} \Omega^k F_0(S^{(k)}\mathcal{C}) \rightarrow \lim_{k \rightarrow \infty} \Omega^k F(S^{(k)}\mathcal{C}) \simeq \Omega F(S\mathcal{C})$$

*is a homotopy equivalence.*

*Proof.* More generally, we show that for all  $n$  the map

$$\sigma: \lim_{k \rightarrow \infty} \Omega^k F_0(S^{(k)}\mathcal{C}) \rightarrow \lim_{k \rightarrow \infty} \Omega^k F_n(S^{(k)}\mathcal{C})$$

given by degeneracies is an equivalence, which implies the result. The map is split by the face maps, sending  $(\alpha_0; \alpha_1, \dots, \alpha_n) \in F_n(S^{(k)}\mathcal{C})$  to  $\delta(\alpha_0; \alpha_1, \dots, \alpha_n) = (\alpha_0 \cdots \alpha_n) \in F_0(S^{(k)}\mathcal{C})$ . We need to show that  $\sigma \circ \delta \sim \text{id}$ .

Let  $X$  be any functor from exact categories to (simplicial) abelian groups satisfying  $X(0) = 0$ . Regarding  $S^{(k)}\mathcal{C}$  as a  $k$  multisimplicial exact category, we see that

$$X(S^{(k)}\mathcal{C} \times S^{(k)}\mathcal{D}) \rightarrow X(S^{(k)}\mathcal{C}) \times X(S^{(k)}\mathcal{D})$$

is  $2k$  connected since the source and target, viewed as  $2k$  multisimplicial groups, agree in total degree less than  $2k$ . This means that under the weakened assumptions Lemma 2.2 should read “ $X S^{(k)} S_2(\mathcal{C}) \rightarrow X S^{(k)}\mathcal{C} \times X S^{(k)}\mathcal{C}$  is  $2k$  connected”, and Lemma 2.5 should read “ $d_0 + d_2 \simeq d_1: \lim_{k \rightarrow \infty} \Omega^k X S^{(k)} S_2 \rightarrow \lim_{k \rightarrow \infty} \Omega^k X S^{(k)}$ ” where  $\Omega$  is a model for the loops within simplicial abelian groups.

Now, letting  $X = F_n$ , we define two natural transformations

$$T_{\text{id}}, T_{\beta}: \lim_{k \rightarrow \infty} \Omega^k F_n S^{(k)} \rightarrow \lim_{k \rightarrow \infty} \Omega^k F_n S^{(k)} S_2$$

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\*Originally published November 1994, **140**, No. 3.

induced by the natural transformations  $t_{\text{id}}, t_{\beta}: F_n \rightarrow F_n S_2$  given by sending an element  $(\alpha_0; \alpha_1, \dots, \alpha_n) \in F_n \mathcal{C}$  to

$$t_{\text{id}}(x) = \left\{ \begin{array}{ccccccccccc} 0 & \longleftarrow & c_p & = & c_p & = & \cdots & = & c_p & \longleftarrow & 0 \\ \downarrow & & \downarrow i_1 & & \downarrow i_1 & & \downarrow i_1 & & \downarrow & & \\ c_p & \xleftarrow{\alpha_0 \pi_2} & c_p \oplus c_0 & \xleftarrow{\text{id} \oplus \alpha_1} & c_p \oplus c_1 & \xleftarrow{\text{id} \oplus \alpha_2} & \cdots & \xleftarrow{\text{id} \oplus \alpha_{p-1}} & c_p \oplus c_{p-1} & \xleftarrow{(\text{id} \oplus \alpha_p) \Delta} & c_p \\ \parallel & & \downarrow \pi_2 & & \downarrow \pi_2 & & \downarrow \pi_2 & & \parallel & & \\ c_p & \xleftarrow{\alpha_0} & c_0 & \xleftarrow{\alpha_1} & c_1 & \xleftarrow{\alpha_2} & \cdots & \xleftarrow{\alpha_{p-1}} & c_{p-1} & \xleftarrow{\alpha_p} & c_p \end{array} \right.$$

and

$$t_{\beta}(x) = \left\{ \begin{array}{ccccccccccc} c_p & \xleftarrow{\beta} & c_p & = & c_p & = & \cdots & = & c_p & = & c_p \\ \parallel & & \downarrow (1 \oplus \beta_1) \Delta & & \downarrow (1 \oplus \beta_2) \Delta & & \downarrow (1 \oplus \beta_p) \Delta & & \parallel & & \\ c_p & \xleftarrow{\alpha_0 \pi_2} & c_p \oplus c_0 & \xleftarrow{\text{id} \oplus \alpha_1} & c_p \oplus c_1 & \xleftarrow{\text{id} \oplus \alpha_2} & \cdots & \xleftarrow{\text{id} \oplus \alpha_{p-1}} & c_p \oplus c_{p-1} & \xleftarrow{(\text{id} \oplus \alpha_p) \Delta} & c_p \\ \downarrow & & \downarrow \beta_{1-1} & & \downarrow \beta_{2-1} & & \downarrow \beta_{p-1} & & \downarrow & & \\ 0 & \longleftarrow & c_0 & \xleftarrow{\alpha_1} & c_1 & \xleftarrow{\alpha_2} & \cdots & \xleftarrow{\alpha_{p-1}} & c_{p-1} & \longleftarrow & 0 \end{array} \right.$$

where  $i_j$  (resp.  $\pi_j$ ) is the  $j^{\text{th}}$  inclusion (resp. projection),  $\Delta$  is the diagonal and  $\beta_k = \prod_{k \leq i \leq q} \alpha_i$ . Note the identities

$$d_0 T_{\text{id}} = \text{id}, \quad d_2 T_{\beta} = \sigma \circ \delta, \quad d_2 T_{\text{id}} = d_0 T_{\beta} = 0 \quad \text{and} \quad d_1 T_{\text{id}} = d_1 T_{\beta}.$$

Hence

$$\text{id} = d_0 T_{\text{id}} \simeq d_1 T_{\text{id}} = d_1 T_{\beta} \simeq d_2 T_{\beta} = \sigma \circ \delta. \quad \square$$

We similarly change Definition 3.2 to

DEFINITION 3.2'. For  $M$  an  $R$  bimodule, we let

$$\text{THH}(R; M) = \lim_{k \rightarrow \infty} \Omega^k \left| \bigoplus_{c \in \text{ob } S^{(k)} \mathcal{C}} \text{Hom}_{S^{(k)} \mathcal{C}}(c, c \otimes_R M) \right|$$

(that is:  $\text{THH}(R, M) = \lim_{k \rightarrow \infty} \Omega^k \text{THH}^{(k)}$  using the notation from the proof of Theorem 3.4); the rest of the argument follows with minor changes as outlined above.

(Received April 12, 1995)