Here is a simple example of a functor that fails to preserve homotopy equivalences. It is taken from a very helpful introduction to model categories by Dwyer and Spalinski.

Let D denote the category $\{a \leftarrow b \rightarrow c\}$, **Top** the category of topological spaces, and $\mathbf{Top}^D$ the category of functors $D \rightarrow \mathbf{Top}$, i.e., pushout diagrams in **Top**. Then we have the functor colim: $\mathbf{Top}^D \rightarrow \mathbf{Top}$ which assigns to each diagram its pushout. It is left adjoint to the functor $\Delta: \mathbf{Top} \rightarrow \mathbf{Top}^D$ which assigns to each pointed space X the constant X-valued diagram. A morphism in $\mathcal{T}^D$ is a the obvious sort of commutative diagram. Consider the morphism

$$\begin{array}{ccccc} D^n & \leftarrow & S^{n-1} & \rightarrow & D^n \\ \downarrow & & \downarrow & & \downarrow \\ * & \leftarrow & S^{n-1} & \rightarrow & * \end{array}$$

in which each vertical map, and hence the morphism in $\mathbf{Top}^D$, is a weak equivalence. However the pushout of the top row (where the two maps are inclusion of the boundary) is S^n, while that of the bottom row is a point. Thus the pushout functor fails to preserve this weak equivalence.

It turns out there is a model structure on $\mathbf{Top}^D$ in which the top row is cofibrant but the bottom row is not, and the pushout functor DOES preserve weak equivalences between cofibrant objects. Let $f: X \rightarrow Y$ be a morphism in $\mathbf{Top}^D$. It consists of three maps $f_a: X_a \rightarrow Y_a$, $f_b: X_b \rightarrow Y_b$ and $f_c: X_c \rightarrow Y_c$.

We define the model structure by saying that f is a weak equivalence/fibration if each of the three maps is, but the definition of a cofibration is more complicated. Let $\partial_b(f) = X_b$ and define $\partial_a(f)$ to be the pushout of

$$\begin{array}{ccc} X_b & \rightarrow & X_a \\ f_b \downarrow & & \downarrow \\ Y_b & \rightarrow & \partial_a(f) \end{array}$$

with a similar definition for $\partial_c(f)$. For each index we get a map $i_*(f): \partial_*(f) \rightarrow Y_*$. We say that f is a cofibration if each of these three maps is. It is a routine exercise (Dwyer-Spalinski Prop.10.6) to verify that this defines a model category structure on $\mathbf{Top}^{\boldsymbol{D}}$.

An object X is cofibrant iff X_b is a CW-complex and the two maps from it are cofibrations. In the example above, the top row is cofibrant but the bottom row is not.

Given a small category J and a model category $\mathcal{C}$, it is not generally clear how to define a model structure on the diagram category $\mathcal{C}^J$. The case of greatest interest to us is $\mathcal{T}_G^{\mathcal{J}_G}$, the category of G-spectra.