$\mathfrak{Cat}_G$

symmetric monoidal category

$\underline{\mathcal{T}_G}$

$(\mathcal{T}^G, \wedge, S^0)$

$(\mathcal{T}, \wedge, S^0)$

$\mathcal{T}^G$

$\mathcal{J}_G$

Examples of closed symmetric monoidal categories:
1. $(Sets, \times, 1)$, sets with Cartesian product.
2. $(\mathcal{T}, \wedge, S^0)$, pointed spaces and smash product.
3. $(\mathcal{T}^G, \wedge, S^0)$, pointed G-spaces with smash product and G-maps.

In each case we can define a category enriched over it. For 1 we have an ordinary category, for 2 a topological category, for 3 a topological G-category (TGC). In each case there is an internal hom, X^Y. Let $\mathfrak{Cat}_G$ denote the category of TGCs. One example of such is $\underline{\mathcal{T}_G}$, the category of pointed G-spaces and nonequivariant maps.

We will define spectra as functors from a certain indexing category to $\underline{\mathcal{T}_G}$.

Def. A.10: The indexing category $\mathcal{J}_G$ is the TGC whose objects are finite dimensional real orthogonal representations V of G. Let $O(V,W)$ denote the space of orthogonal embeddings $V \to W$. It is called a Stiefel manifold. It is empty if $\dim W < \dim V$. For each such embedding we have an orthogonal complement $W - V$, giving us a vector bundle over $O(V,W)$. Let $\mathcal{J}_G(V,W)$ be its Thom space, which is a pointed G-space.

When dimW<dim V, $O(V,W)$ is empty and $\mathcal{J}_G(V,W)$ is a point. Informally, $\mathcal{J}_G(V,W)$ is a wedge of spheres S^{W-V} parametrized by the orthogonal embeddings $V \to W$.

Def. A.13: An orthogonal G-spectrum X is a functor $\mathcal{J}_G \to \underline{\mathcal{T}_G}$.

Informally we have a G-space X_V for each representation V, and for each embedding $V \to W$ a map $S^{W-V} \wedge X_V \to X_W$. Here S^V denotes the one point compactification of V, which is G-space with ∞ as base point.

In the original definition of a spectrum we had for each n a space X_n and a map $\Sigma X_n = S^1 \wedge X_n \to X_{n+1}$. Now we have a family of such maps parametrized by the orthogonal embeddings $R^n \to R^{n+1}$.