Recall E denotes the mod 2 Eilenberg-Mac Lane spectrum, A denotes the mod 2 Steenrod algebra and H^*X (H_*X) denotes the mod 2 cohomology (homology) of X.

$H^nX = \pi_n E \wedge X$ *and* $H^nX = [X, \Sigma^n E]$

$H^*E = A$
There is a unit map $f: S^0 \to E$ inducing surjections in both H^* and π_*. We can smash this map with any spectrum X and get a map
$f: X \to X \wedge E$ which is surjective in cphomology but not necessarily in homotopy.

For a locally finite wedge K of suspensions of E,
$\pi_* K = Hom_A(H^*K, Z/2)$.

Some basic facts about these are listed in Prop 2.1.2 of the green book, see
http://www.math.rochester.edu/people/faculty/doug/mybooks/ravenel2.pdf.

Consider the long exact sequence (2.1.4). Applying the functor
$Hom_A(\,*\,, Z/2)$ to each H^*K_s yields a cohcain complex of the form
$\pi_* K_0 \to \pi_* \Sigma K_1 \to \pi_* \Sigma^2 K_2 \to \cdots$
Its cohomology is $Ext_A^{s,t}(H^*X, Z/2)$. This is $E_2^{s,t}$ of a spectral sequence converging to $\pi_{t-s}X$. The indxeing is such that
d_r: $E_r^{s,t} \to E_r^{s+r,t+r-1}$. This differential raises s by r and decreases the topological dimension $t-s$ by 1. When $X = S^0$, we have
$E_2 = Ext_A^{s,t}(Z/2, Z/2)$. Finding it is a purely algebraic problem suitable for a computer.

Isaksen's chart can be found at
http://www.math.rochester.edu/people/faculty/doug/otherpapers/isaksen-charts.pdf

0-line ($s = 0$, x-axis):
$E_2^{0,t} = Hom_A(k, \Sigma^t k)$ where $k = Z/2$.

Digression in homological algebra:
$Ext_R^s(M, N)$ is the set of equivalence classes of exacts sequences of of R-modules of length $s+2$ starting with N and ending with M. See any book on homological algebra.

1-line ($s = 1$)

$$E_2^{1,t} = Ext_A^1(k, \Sigma^t k)$$

We are looking for a short exact sequence of A-modules of the form $0 \to \Sigma^t k \to M \to k \to 0$. The top class in dimension t must be Sq^t on the 0-dimensional class. The Adme relation implies that Sq^t can be written as a sum of products of lower operations unless $t = 2^j$ for some j. For example $Sq^3 = Sq^1 Sq^2$. When $t = 2^j$, we get a nontrival element in $Ext_A^1(k, \Sigma^t k)$ denoted by $h_j \in E_2^{1,2^j}$. These are permanent cycles for $0 \le j \le 3$. They reprsent elements in $\pi_{2^j-1} S^0$, namely 2ι (where $\iota \in \pi_0 S^0$ denmotes the identity map) for $j = 0$, and the supensions of the Hopf maps for $j > 0$. Recall there are maps of spaces constrcuted by Hopf in 1930, $S^3 \to S^2, S^7 \to S^4$, and $S^{15} \to S^8$.

What about h_4? Is there a similar map $S^{31} \to S^{16}$? This would be related to a division algebra over the reals of dimension 16. Adams proved that the map does not exist, which implies that the diovision algebra dioes not exist.