7/8/09

Commentary on Akhmetiev's main paper, partially translated by Landweber

Here is a list of sections:

- **Section 1. Self-intersections of immersions and the Kervaire invariant**, page 1.

  An outline of the program is given, similar to my synopsis above, but with more detail. There is a diagram on page 7 showing subgroups of interest in $\mathbf{Z}/2^{[d]}$ (the 2-Sylow subgroup of the symmetric group on 2^d letters) for $2 \leq d \leq 6$.

- **Section 2. Geometric control of the manifold of self-intersections of a skew-framed immersion**, page 8.

  Definition 3 says that a skew framed immersion is *abelian* if the structure group for the double point manifold can be reduced from $\mathbf{Z}/2^{[2]} = D_4$ to $(\mathbf{Z}/2)^2$. Definition 4 says that it *has an abelian structure* if it satisfies a milder form of this condition. Definition 5 is that of a compression of order q.

  Theorem 6 says that an immersion in $\mathbf{R}^n$ of a certain small codimension k has an abelian structure if it has a compression of a certain order q. The simplest case is $(n, k, q) = (2^{j+1} - 2, 2^{j-4} - 2, 32)$ for $j \geq 6$. However, we know that in order to insure the existence of an order 32 compression, j must be much larger. This is the first of several theorems saying that a certain compression guarantees the existence of som,e kind of gemotric structure on the immersion.

  Lemma 7 says something about a map of odd codimension of a projective space into Euclidean space. The content appears to me to be trivial and the reference given for its proof seems to be incorrect.

  Theorem 6 is derived from Lemma 7 in a 2 page argument that I am unable to follow.

- **Section 3. Bicyclic structure on a $\mathbf{Z}/2^{[3]}$-immersion**, page 14.

  Subgroups of $\mathbf{Z}/2^{[3]}$ and $\mathbf{Z}/2^{[4]}$ isomorphic to $\mathbf{Z}/4 \times \mathbf{Z}/2$ and $\mathbf{Z}/4 \times \mathbf{Z}/4$ respectively are specified. Definition 8 says that a $\mathbf{Z}/2^{[3]}$-framed ($\mathbf{Z}/2^{[4]}$-framed) immersion is $(\mathbf{Z}/4 \times \mathbf{Z}/2)$-framed ($(\mathbf{Z}/4 \times \mathbf{Z}/4)$-framed) if the structure group for the immersion itself (rather than for its immersion of double points as in the previous section) can be reduced to the indicated subgroup.

  Definitions 9 and 10 refer to the double point immersions associated with $\mathbf{Z}/2^{[2]}$-framed and $\mathbf{Z}/2^{[3]}$-framed immersions. They are said to *have G-structures* (for the subgroups G of $\mathbf{Z}/2^{[3]}$ and $\mathbf{Z}/2^{[4]}$ above) if their Kervaire invariants satisfy certain conditions similar to that in Definition 4. The latter structure is also called a *bicyclic structure*.

  I am unable to decipher the staements of Theorems 11 and 12, whose proofs have yet to be translated. Corollary 13 says that under the hypotheses of Theorem 6 (namely a compression condition), a $\mathbf{Z}/2^{[3]}$-framed immersion has a bicyclic structure.

- **Section 4. Biquaternionic structure on a $\mathbf{Z}/2^{[5]}$-immersion**, page 19.

  This section is very similar to the previous one. Subgroups of $\mathbf{Z}/2^{[5]}$ and $\mathbf{Z}/2^{[6]}$ isomorphic to $Q_8 \times \mathbf{Z}/4$ (where Q_8 denote the quaternion group) and $Q_8 \times Q_8$ respectively are specified. Definitions 14, 15 and 16 are analogous to 8, 9 and 10. Theorems 17 and 18 are similar to Theorems 11 and 12 and have untranslated proofs. Corollary 19 says that under the hypotheses of Theorem 6, a $\mathbf{Z}/2^{[5]}$-framed immersion has a biquaternionic structure.

- **Section 5. Solution of the Kervaire invariant problem**, page 24.

  The Main Theorem states that the Kervaire invariant of a skew framed immersion into $\mathbf{R}^{2^l-2}$ vanishes for $l \gg 0$. Results stated earlier imply that such an immersion leads to a $\mathbf{Z}/2^{[5]}$-framed immersion with biquaternionic structure. The last two pages of this section is a calculation aimed at showing that a certain characteristic number vanishes, which would make the Kervaire invariant vanish. There is a similar calculation on the last few pages of his Princeton slides. I have not had time to study this carefully.

- **Section 6. Proof of Theorems 11 and 12** [in Section 3], page 28, not yet translated. It is 15 pages in the Russian original. Hopefully this section and the next will be translated soon by Akhmet'ev.

- **Section 7. Proof of Theorems 17 and 18** [in section 4], page 28, not yet translated. It is 19 pages in the Russian original.

- **Section 8. The compression theorem**, page 28. Only the statement is translated. It is 23 pages in the Russian original.

- **References**, page 28.