Toward higher chromatic analogs of elliptic cohomology and topological modular forms

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- L_0 is rationalization. Rational stable homotopy theory is very well understood. It detects only the 0-stem in the stable homotopy groups of spheres.

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- L_1 is localization with respect to real or complex K-theory. It detects the image of J and the α family in the stable homotopy groups of spheres. The Lichtenbaum-Quillen conjecture is a statement about L_1 of algebraic K-theory.

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- L_2 is localization with respect to elliptic cohomology [LRS95] or the theory of topological modular forms of Hopkins *et al*. It detects the β family in the stable homotopy groups of spheres. Davis’ nonimmersion theorem for real projective spaces was proved using related methods.

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$$L_0\mathcal{S} \longleftarrow L_1\mathcal{S} \longleftarrow L_2\mathcal{S} \longleftarrow L_3\mathcal{S} \longleftarrow \cdots \longleftarrow \mathcal{S}_{(p)}$$

- For $n > 2$ there is no comparable geometric definition of L_n, which can only be constructed by less illuminating algebraic methods related to BP-theory. It detects higher Greek letter families in the stable homotopy groups of spheres. The nth Morava K-theory is closely related to it.

The m-series of a formal group law

Definition 1 *Let F be 1-dimensional formal group law. For a positive integer m, the m-series is defined inductively by*

$$[m]_F(x) = F(x, [m-1]_F(x))$$

where $[1]_F(x) = x$.

Examples of m-series

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- For $F(x,y) = \frac{x+y}{1+xy}$, we have

$$\begin{aligned}[m](x) &= \textstyle\sum_i \binom{m}{2i+1} x^{2i+1} \Big/ \sum_i \binom{m}{2i} x^{2i} \\ &= \frac{mx + \binom{m}{3} x^3 + \ldots}{1 + \binom{m}{2} x^2 + \ldots}\end{aligned}$$

The height of a formal group law

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Definition 3 *The* **height** *of F is the integer n. If $[p]_F(x) = 0$ (which happens when $F(x, y) = x + y$), the height is defined to be ∞.*

Examples of heights

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- The formal group law associated with an elliptic curve is known to have height at most 2.
- v_n-periodic phenomena (the nth layer in the chromatic tower) are related to formal group laws of height n.

Question

How can we attach formal group laws of height > 2 to geometric objects (such as algebraic curves) and use them get insight into cohomology theories that go deeper into the chromatic tower?

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If θ is Landweber exact [Lan76], then we get an MU-module spectrum E with $\pi_*(E) = R$.

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Note that a 1-dimensional summand of the formal completion $\widehat{J}(C)$ is *not* the same thing as 1-dimensional factor of the Jacobian $J(C)$.
The latter would be an elliptic curve, whose formal completion can have height at most 2.
There is a theorem that says if an abelian variety A has a 1-dimensional formal summand of height n for $n > 2$, then the dimension of A (and the genus of the curve, if A is a Jacobian) is at least n.

Artin-Schreier curves

Theorem 4 *(2002) Let* $C(p, f)$ *be the curve over* $\mathbf{F}_p$ *defined by the affine equation*

$$y^e = x^p - x \qquad \textit{where } e = p^f - 1.$$

(Assume that $f > 1$ *when* $p = 2$*.) Then its Jacobian has a 1-dimensional formal summand of height* $(p-1)f$*.*

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(Assume that $f > 1$ *when* $p = 2$*.) Then its Jacobian has a 1-dimensional formal summand of height* $(p-1)f$.

The resulting genus is *not* Landweber exact, so this does not lead to a cohomology theory.

Properties of the curve $C(p, f)$

- Its genus is $(p-1)(p^f-2)/2$. (Thus it is zero in the excluded case $(p, f) = (2, 1)$.)

Properties of the curve $C(p, f)$

- It has an action of the group

$$G = \mathbf{F}_p \rtimes \mu_m, \quad \text{where } m = (p-1)(p^f - 1)$$

and μ_m is the group of mth roots of unity, given by

$$(x, y) \mapsto (\zeta^{p^f - 1} x + a, \zeta y)$$

for $a \in \mathbf{F}_p$ and $\zeta \in \mu_m$.

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for $a \in \mathbf{F}_p$ and $\zeta \in \mu_m$.
This group is isomorphic to a maximal finite subgroup of the hth Morava stabilizer group, and it acts appropriately on the 1-dimensional formal summand of $\widehat{J}(C(p, f))$.

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- $C(2,3)$ has genus 3 and its Jacobian has a 1-dimensional formal summand of height 3.
- $C(2,4)$ and $C(3,2)$ each have genus 7 and their Jacobians each have a 1-dimensional formal summand of height 4.

Remarks

- Theorem 4 was known to and cited by Manin in 1963 [Man63]. Most of what is needed for the proof can be found in Katz's 1979 Bombay Colloquium paper [Kat81] and in Koblitz' Hanoi notes [Kob80]. (The latter book has been googled and is available online.)

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- The original proof rests on the determination of the zeta function of the curve by Hasse-Davenport in 1934 [HD34], and on some properties of Gauss sums proved by Stickelberger in 1890 [Sti90]. The method leads to complete determination of $\widehat{J}(C(p, f))$.

More remarks

We have reproved Theorem 4 using Honda's theory of commutative formal group laws developed in the early '70s in [Hon70] and [Hon73]. (These papers are also available online.)

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This proof does not rely on knowledge of the zeta function and can be modified to prove Theorem 5 below.

Deforming the Artin-Schreier curve

We want a lifting of $C(p, f)$ to characteristic 0 that admits a coordinate change similar to the one for the Weierstrass curve used in the construction of tmf.

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with (nonaffine) coordinate change fixing y and sending

$$x \;\mapsto\; x + \sum_{i=1}^{f} t_i y^{(p^f - p^i)/p}.$$

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$$x \mapsto x + \sum_{i=1}^{f} t_i y^{(p^f - p^i)/p}.$$

The t_i above are related to the generators of the same name in $BP_*(BP)$.

Deforming the Artin-Schreier curve

In order to state this precisely we need some notation. Let

$$I = (i_1, i_2, \ldots i_f)$$

be an f-tuple of nonnegative integers and define

$$|I| = \sum_k i_k \qquad ||I|| = \sum_k (p^k - 1) i_k$$

$$t^I = \prod_k t_k^{i_k} \qquad I! = \prod_k i_k!$$

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The coefficients in our equation will be formal variables a_I with $|I| \leq p$ (where $a_0 = p!$) with topological dimension $2||I||$.

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Deforming the Artin-Schreier curve

Then the equation for our deformed curve is

$$\begin{aligned} y^e &= \sum_{i=0}^{p} \frac{x^{p-i}}{(p-i)!} \sum_{|I|=i} a_I y^{(ei-||I||)/p} \\ &= x^p + a_m x + \ldots \end{aligned}$$

where (as before) $e = p^f - 1$ and $m = (p-1)e$.

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where (as before) $e = p^f - 1$ and $m = (p-1)e$. The effect of the coordinate change on the coefficients a_I is given by

$$a_I \mapsto \sum_{J+K=I} a_J \frac{t^K}{K!}.$$

An infinitesimal deformation

Theorem 5 (2004) *Let*

$$\begin{aligned} A &= \mathbf{Z}_p[a_I : 0 < |I| \leq p] \\ \overline{A} &= A/(a_m - 1) \\ \overline{A} \supset J &= (a_i : i \neq m,) \end{aligned}$$

Then the Jacobian of curve above defined above over the ring $\overline{A}/J^2$ has a 1-dimensional formal summand of height h. The corresponding formal group law has Landweber exact liftings to $\overline{A}$ and $a_m^{-1}A$.

An infinitesimal deformation

The map $BP_* \to \overline{A}$ is given by

$$v_r = \begin{cases} pa_{m+p^r-1} + a_{p^r-1} & \text{if } 1 \le r \le \min(f, h-1) \\ a_{se+p^i-1} & \text{if } f < r < h \text{ and } p > 2 \\ m - 2a_{2e} & \text{if } r = h \text{ and } p = 2 \\ 1 & \text{if } r = h \text{ and } p > 2 \end{cases}$$

up to unit scalar, where $r = sf + i$ with $1 \le i \le f$.

A fantasy

There is an associated Hopf algebroid

$$\Gamma = A[t_1, \ldots, t_f]$$

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Conjecture 6 *For each (p, f) as above there is a spectrum generalizing* tmf *whose homotopy can be computed by an Adams-Novikov type spectral sequence with*

$$E_2 = \mathrm{Ext}_\Gamma(A, A).$$

A is too big

This conjecture is not likely to be true for $f > 1$ because the ring A is too large. Ideally its Krull dimension should be pf, the sum of the height of the formal group law and the number of coordinate change parameters.

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The Krull dimension of A is $\binom{p+f}{f} - 1$. For $f = 2$ this is $p(p+3)/2$ instead of the desired $2p$.

A smaller ring R

Replace the equation above with

$$y^e = \prod_{j=1}^{p} \left(x + \sum_{i=1}^{f} r_{i,j} y^{(p^f - p^i)/p} \right)$$

with $|r_{i,j}| = 2(p^i - 1)$.

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with $|r_{i,j}| = 2(p^i - 1)$. Thus we get a curve defined over the ring

$$R = \mathbf{Z}_p[r_{i,j} : 1 \leq i \leq f,\ 1 \leq j \leq p],$$

which has the desired Krull dimension.

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and

$$\mathrm{Ext}^s_\Gamma(R) = \begin{cases} \mathbf{Z}_p[r_{i,j} - r_{p,i} \colon 1 \leq j \leq p-1] & \text{for } s = 0 \\ 0 & \text{for } s > 0. \end{cases}$$

A slightly smaller ring B

The equation for the curve is actually defined over the subring

$$B = R^{\Sigma_p} = \mathbf{Z}_p[r_{i,j} : 1 \leq i \leq f,\, 1 \leq j \leq p]^{\Sigma_p}$$

where Σ_p acts on R via the second subscript.

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where Σ_p acts on R via the second subscript. This ring is a quotient of A, but its structure appears to be unknown for $f > 1$ except for $p = 2$. B is clearly a module (presumably free of rank $p!^{f-1}$) over the subring

$$C = R^{\Sigma_p^f}$$

where the f copies of Σ_p act independently on the f sets of p generators of R.

The ring C

The structure of C is well known, namely

$$C = \mathbf{Z}_p[\sigma_{i,k} : 1 \leq i \leq f,\ 1 \leq k \leq p]$$

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$\sigma_{i,k}$ is the image of $a_{k(p^i-1)}/(p-k)!$.

Relation to tmf **at** $p = 2$

For $(p, f) = (2, 2)$ our equation reads

$$y^3 = x^2 + (a_1 y + a_3)x + a_2 y^2 + a_4 y + a_6,$$

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so our a_is are the Weierstrass a_is up to sign. In the ring B there is a relation

$$(2a_4 - a_1 a_3)^2 = (4a_2 - a_1^2)(4a_6 - a_3^2),$$

which makes it a free module on $\{1, a_4\}$ over the ring

$$C = \mathbf{Z}_2[a_1, a_2, a_3, a_6].$$

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It seems likely that our conjecture (with A replaced by B) would lead to the spectrum

$$tmf \wedge \left(S^0 \cup_\nu e^4\right).$$

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