6/11/09

Letter on an Akhmet'ev paper

Dear Akhmetiev fans,

Andrew Ranicki just sent me a paper by Aklmetiev entitled "GEOMETRIC APPROACH TO STABLE HOMOTOPY GROUPS OF SPHERES. KERVAIRE INVARIANTS. II," which is now posted on my website. It is a 16 page English translation of a 24 page paper that appeared in 2007. The most interesting part may be the following conjecture, which is similar to something he claimed to have proved in his talk in Princeton last month. (The slides for that talk are on my website.)

Conjecture. For an arbitrary $q > 0$, $q = 2 \bmod 4$, there exists a positive integer $l_0 = l_0(q)$ such that for an arbitrary $n = 2^l - 2$, $l > l_0$, an arbitrary element $a \in Imm^{sf}(3n + q/4, n - q/4)$ is stably regular cobordant to a stably skew-framed immersion with I_b-type of self-intersection.

To paraphrase this, given q as above and $n = 2^l - 2$ for $l \gg 0$, consider a skew framed immersion of a manifold M^{n-k} in $\mathbf{R}^n$ where the codimension k is chosen so that the manifold L^{n-4k} of quadruple points has dimension q. "Skew framed" means that the normal bundle comes equipped with an isomorphism to a direct sum of k copies of a line bundle κ, so its structure group is $Z/2$. This means that the normal bundle of the immersed manifold N^{n-2k} of double points is isomorphic to a direct sum of k copies of a 2-plane bundle η with structure group D_4, the dihedral group of order 8. The conjecture says that if l is large enough, this structure group can be reduced to a certain noncyclic subgroup of index 2.

The line bundle κ over M^{n-k} is classified by a map to $\mathbf{R}P^\infty$. Theorem 1 says the conjecture holds when this map compresses to $\mathbf{R}P^{n-k-1-q}$. It has an intricate 5 page proof which I have not read.

The quadruple point manifold L^q has a normal bundle isomorphic to k copies of a 4-plane bundle ζ with structure group $\mathbf{Z}/2 \wr D_4$, which is the 2-Sylow subgroup of the symmetric group Σ_8. In his Princeton slides he denotes this group by $Z/2^{[3]}$ and D_4 (the 2-Sylow subgroup of Σ_4) by $Z/2^{[2]}$. Here he describes a certain subgroup I_4 isomorphic to $Z/2 \oplus Z/4$.

Theorem 2, which also has an intricate 5 page proof that I have not read, says that when $q = 62$, $l \geq 15$ and the hypothesis of Theorem 1 is met, then the structure group for ζ can be reduced from $\mathbf{Z}/2^{[3]}$ to the abelian subgroup I_4.