Errata for *Complex cobordism and stable homotopy groups of spheres* by Douglas C. Ravenel. Special thanks go to Peter Landweber, Nori Minami, Igor Kriz, Hirofumi Nakai and Bill Richter. Special attention should be paid to the changes in §4.3. Last updated July 31, 2003.

General comments:

A list of figures and tables at the beginning would be desirable.

The proof of Theorem 2.3.3 may be incorrect.

Misprints:

Page vii, 1.2: $\pi_*(S^n)$

Page ix, 3.5: $MSp...v_n$-periodicity

Page ix, 4.2: $BP\langle n\rangle$

Page xi, 6.3: $H^*(S(n))$

Page xvi, lines -8 and -20: appendices

Page xvi, final paragraph, add: Peter Landweber was kind enough to provide us with a copious list of misprints he found in the first edition. Nori Minami and Igor Kriz helped in correcting some errors in §4.3.

Page xvii, line -2: the Troisième Cycle

Chapter 1

Page 3, line -6: The groups $\pi_{n+k}(S^n)$ are called stable if

Page 3, last full sentence: Most of the time we will not be concerned with unstable groups.

Page 4, line 1: See the tables in Appendix 3, along with Theorem 1.1.13.

Page 4, table: π_8^S should be $(\mathbf{Z}/2)^2$, π_{11}^S should be $\mathbf{Z}/2 \oplus \mathbf{Z}/504$.

Page 4,line -1:$(-1)^{ij}$

Page 5, line -13: We define $\hat{f} \ldots$

Page 5, line -11: boundary S^{n-1}

Page 5, line -7: define $\tilde{f} : S^{n+i} \to S^n$

Page 6, insert at start of third paragraph: The image of J is also known to be a direct summand; a proof can be found for example at the end of Chapter 19 of Switzer [1].

Page 6, line -3: $\in \pi_{q-1}^S$

Page 6, line -1: $\alpha_1\beta_2 \in \pi_{(2p+2)q-3}^S$

Page 8, lines -1 and -2: replace d_2 by d_3

Page 9, omit period in line above 1.2.6.

Page 9, no new paragraph in line below 1.2.6.

Page 10, 3 lines above 1.2.11: β and $\mathcal{P}^{p^i}$

Page 11, 3 lines below 1.2.14: Serre's

Page 11, bottom paragraph, insert after fourth sentence: These extra elements appear in the chart to the right of where they should be, and the lines meeting them should be vertical.

Page 12: Four d_2's have been left out. Their targets are at $(39,9)$, $(42,9)$, $(42,10)$ and $(45,10)$. The last of these is supported by an element at $(46,8)$ which should be connected to the element at $(43,7)$ by a line representing h_0-multiplication.

Page 12: spectral sequence

Page 13, line 8: 31 differentials

Page 13: line -2: bordant

Page 14, lines 10 and 14: complex bordism ring

Page 15: spectral sequence

Page 16, line -1: $[[x \otimes 1, 1 \otimes x]]$

Page 17, 1.3.3: map μ :

Page 18: 1.3.5: π_*^S

Page 19, 1.3.9: $H^0(G; L/(p))$

Page 21, 3 lines above 1.3.19: Hence G-invariant prime ideals

Page 24, line -18: metaphor

Page 25, lines 5–6 after 1.4.2: $[1]_F(x) = x$

Page 25, 1.4.3 (a): closure of $\mathbf{F_p}$

Page 25, 1.4.3 (b): ...as in 1.3.16, where K is a finite field.

Page 25, line -9: gives a topological basis

Page 26, line 13: chromatic spectral sequence

Page 26, line -5 should not be in italics.

Page 27, line -17: 1.3.17 and 1.3.19.

Page 27, line -11: $E_2^{n,0} \to E_\infty^{n,0}$

Page 28, line 17: dimensions not divisible

Page 28, line 20 should end with a period.

Page 30, 1.5.8 (d): $E_1^{k,2m} = 0$ for $k < qm - 1$

Page 31, line -12: an element α

Page 33, line 8: the subgroup and cokernel

Page 33, line 14: $\pi_7(S^3) = E_1^{5,2} = \mathbf{Z}/(2)$

Page 33, fifth line before the diagram 1.5.10: The fibration 1.5.1 can be looped

Page 33, third line of 1.5.12 (a): $E_\infty^{k,n}$ is the subquotient

Page 35, 1.5.14: The middle right square of this diagram only commmutes after a single looping. This blemish does not affect calculations of homotopy groups.

Page 36, 1.5.17: (Kambe, Matsunaga and Toda [1])

Page 37, line 1: $E_r^{k-1,n-r}$....$\mathbf{R}P_{n-r-1}^{n-1}$.

Page 37, line 4: $i = [r/2]$.

Page 38, line 7 above 1.5.20: Priddy [2]

Page 40, line 4 above 1.5.23: α_j in $\pi_*(J)$

Page 40, add to 1.5.23 (a): (We will denote the generator of $E_1^{k,k+1}$ by x_k and the generator of $E_1^{k,k+1+m}$ for $m > 0$ by the name of the corresponding element in $\pi_m(J)$.)

Page 44, line 6 after 1.5.29:... for $j > 0$.

Page 46, line -3: $M(\overline{\alpha}_3) \ni \overline{\alpha}_3^2 = \theta_3$.

Chapter 2

Page 50, 3 lines below 2.1.6: $j_1 = \pi_{t-s}(f_s)$

Page 50, 6 lines below 2.1.6: ker $j_1 =$

Page 51, line 4: This group will be identified (2.1.12)

Page 51, last labelled vertical arrow in Figure 2.1.9: $\pi_{u-1}(g_{s+1})$

Page 51, third line after the Figure 2.1.9: $d_1^{s,t} = (\pi_{u-1}(f_{s+1}))(\partial_{s,u})$

Page 55, line 2: (X_s, g_s)

Page 55, third line before end of proof of 2.1.16: Each X_s also satisfies the hypotheses of the lemma, so we conclude ...

Page 58, line 3: such that $h_s(E \wedge f_s)$ is an identity map of $E \wedge X_s$.

Page 58, line -2: for all r

Page 62, third line of the proof of 2.2.14: $X_s = \overline{E}^{(s)} \wedge X$

Page 67, line 1: Let W' be

Page 67, line 3: Since Σfh

Chapter 3

Page 70, third line of 3.1.1 (b): $|\tau_n| = 2p^n - 1$

Page 70, fourth line of 3.1.1 (b): $\Delta\tau_n = \tau_n \otimes 1 + \sum_{0 \le i \le n} \xi_{n-i}^{p^i} \otimes \tau_i$

Page 70, add to 3.1.1 (c): $c(\xi_0) = 1$

Page 70, 3.1.1 (c) line 1: For each p

Page 73, fifth line of the proof of 3.1.9: $\mathrm{Ext}_{\Gamma_i}(K, K) = P(y_i)$

Page 73, 3.1.10, line 2: as in 3.1.7

Page 76, 3.1.18: $\mathrm{Ext}_{A(1)_*}(\mathbf{Z}/(2), \mathbf{Z}/(2))$

Page 77, line above 3.1.24: implies 3.1.23.

Page 80, line -4: $x^p = \xi(x)$

Page 86, 3.2.12: the 2-component of

Page 91, 3.3.7(b): 0 for $n > m$

Page 93, second to last paragraph: It is also true that 300 is cohomologous in Λ to 111, the difference being the coboundary of $40 + 22$.

Page 94: [Remove lines under 4111 and 24111 at bottom of figure and elsewhere. The elements 1 and 11 in the 11-stem for $n = 10$ should be interchanged.]

Page 98, line 3 of 3.3.17: For the reader's amusement

Page 98, paragraph after 3.3.17: [Replace last three sentences with] Vertical and diagonal lines as usual represent right multiplication by λ_0 and λ_1, i.e., by h_0 and h_0 respectively. This point is somewhat delicate. For example the element with in the 9-stem with filtration 4 has leading term (according to 3.3.10) 1233, not 2331. However these elements are cohomologous, their difference being the coboundary of 235.

Page 100, line 2 of Section 4: we comment on the status

Pages 100–101, Theorem 3.4.2(c): spanned by $\{h_i h_j : 0 \le i < j-1\}$, a_0^2, ... $k_i = \ldots \in \mathrm{Ext}^{2,(2p+1)p^i q}$

Page 104, line 3 of Proof of 3.4.9: $(p^n q - 1)$-connected

Page 111, line 3 of fourth paragraph: Bahri and Mahowald [1]

Page 112, replace 3.5.2 with the following: For each $n \ge 0$, A_* has a decreasing filtration (A1.3.5) $\{F^s A_*\}$ where F^s is the smallest possible subgroup satisfying $\bar{\xi}_i^{2^j} \in F^{2^{i+j-n-1}-1}$ for $j \le n+1$.

Page 112, insert after 3.5.2: In particular, $F^0/F^1 = A(n)_*$, so $A(n)_* \subset E_0 A_*$ where

$$A(n)_* = A_*/(\bar{\xi}_1^{2^{n+1}}, \bar{\xi}_2^{2^n}, \ldots, \bar{\xi}_n^2, \bar{\xi}_{n+1}\bar{\xi}_{n+2}, \ldots).$$

We also have $\bar{\xi}_i^{2^j} \in F^{2^{j-n-1}(2^i-1)}$ for $j \ge n+1$. Hence there is a spectral sequence (A1.3.9) converging to $\mathrm{Ext}_{A_*}(\mathbf{Z}/(2), M)$ with $E_1^{s,t,u} = \mathrm{Ext}^{s,t}_{E_0 A_*}(\mathbf{Z}/(2), E_0 M)$ and $d_r\colon E_r^{s,t,u} \to E_r^{s+1,t,u+r}$, where the third grading is that given by the filtration, M is any A_*-comodule, and $E_0 M$ is the associated $E_0 A_*$-comodule (A1.3.7).

Page 113, Lemma 3.5.10: [Insert extra right parenthesis at end of displayed formula.]

Page 117, line 18: and Tangora [1], Tangora [5] and Bruner [2].

Page 117, line 22: See also Milgram [2], Kahn [2], Bruner *et al* [1] and Makinen [1].

Page 117, line after displayed formula$A$-module $\mathbf{Z}/(2)[x, x^{-1}]$

Chapter 4

Page 120, line 18: The pullback of γ_{n+1} under this map

Page 121, 4.1.1: associative commutative ring spectrum

Page 121, 4.1.3, line 5: gives a complex orientation

Page 122, line 5: multiple of x_H

Page 123, 4.1.8: Let E be a complex oriented ring spectrum.

Page 125, proof of 4.1.11 lines 4 and 5: These give complex orientations

Page 126, line 4: (2.2.12) such that the map $g : MU_{(p)} \to BP$ is multiplicative,

Page 126, line 5: $\mathbf{Q}[g_*(m_{p^k-1}) : k > 0]$ with $g_*(m_n) = 0$ for $n \neq p^k - 1$;

Pgaes 128–129, statement of 4.1.18 should be italicized.

Page 130, line -3: iff G has periodic cohomology.

Page 132, line 5: Mironov

Page 132, lines 12–13: construction) by killing $(v_{n+1}, v_{n+2}, \ldots)$

Page 132, lines 14: $H_*(BP\langle n\rangle, \mathbf{Z}/(p))$

Page 133, line 2: onto iff Hom

Page 133, line 7: $A/(Q_n)_*$

Page 135, lines 17–19: The exact functor theorem can be formulated globally in terms of MU-theory and $\pi_*(K)$ [viewed as a $\pi_*(MU)$-module via the Todd genus $td : \pi_*(MU) \to \mathbf{Z}$] satisfies the hypotheses.

Page 135, third paragraph, last sentence: Using similar methods they were able to show that real K-theory is determined by symplectic cobordism.

Page 136, line -18: any complex oriented (4.1.1)

Page 139, Proof of 4.3.4, line 2: $x = y + pb + \sum a_i$

Page 143, 4.3.14: In $BP_*(BP) \otimes_{BP_*} BP_*(BP)$ let $b_{i,j} = w_{j+1}(\Delta_i)$. [Delete second sentence.]

Page 143, paragraph above 4.3.16: Now we will simplify the right unit formula 4.3.1. First we need a lemma.

Pages 143–144, 4.3.16, second expression in the first formula:

$$\sum_{i,|I|\geq 0}{}^F[(-1)^{|I|}](t_i(t_I)^{p^i})$$

[Omit the second equation (the one involving $c(t_i)$) from the statement, and the last sentence (including the displayed formula) from the proof.]

Page 144, paragraph after 4.3.16:

Now we need to use the conjugate formal group law $c(F)$ over $BP_*(BP)$, defined by the homomorphism $\eta_R : BP_* \to BP_*(BP)$. Its logarithm is

$$\log_{c(F)}(x) = \sum_{i\geq 0} \eta_R(\lambda_i)x^{p^i} = \sum_{i,j\geq 0} \lambda_i t_j^{p^i} x^{p^{i+j}}.$$

An analog of 4.3.9 holds for $c(F)$ with v_I replaced by $\eta_R(v_I)$.

The last equation in the proof of A2.2.5 reads

$$\sum \lambda_i v_j^{p^i} t_k^{p^{i+j}} = \sum \lambda_i t_j^{p^i} \eta_R(v_k)^{p^{i+j}} = \sum \eta_R(\lambda_i)\eta_R(v_j)^{p^i}$$

while 4.3.16 gives

$$\sum \lambda_i = \sum (-1)^{|K|} \lambda_i t_j^{p^i} t_K^{p^{i+j}}.$$

Combining these and reindexing gives

$$\sum (-1)^{|J|} \eta_R(\lambda_i)(t_J(v_k t_l^{p^k})^{p^{||J||}})^{p^i} = \sum \eta_R(\lambda_i)\eta_R(v_j)^{p^i},$$

which is equivalent to

4.3.17 $$\sum_{i\geq 0}{}^{c(F)} \eta_R(v_i) = \sum_{|I|,j,k\geq 0}{}^{c(F)} [(-1)^{|I|}]_{c(F)} (t_I(v_j t_k^{p^j})^{p^{||I||}}).$$

Page 144, line 2 above 4.3.18:

$$R_n = N_n \cup \bigcup_{\substack{||J||=i \\ 0<i<n}} \{\eta_R(v_J) w_J(R_{n-i})\}$$

Page 144, 4.3.20 equation:

$$\sum_{0\leq i\leq k} v_{n+i} t_{k-i}^{p^{n+i}} - \eta_R(v_{n+k-i})^{p^i} t_i = \sum_{0\leq j\leq k-n-1} v_{n+j} c_{k-j,n+j}.$$

Page 145, 4.3.21: Use italics in statement of Corollary.
Page 145, first line after 4.3.21: $c_{i,j}$
Page 145, 4.3.22(b): $d(c_{n+i,j+1})$
Page 149, Proof of 4.3.8, line 3: $w_{K'} \equiv w_K^p$
Page 149, Proof of 4.3.8, line 8:

$$\sum_{IJ=K} \frac{\Pi(K)}{\Pi(I)} w_J^{p^{||I||}} = \sum_{IJ=K''} \frac{\Pi(K'')}{\Pi(I)} w_J^{p^{||I||}}.$$

Page 150, first sentence: By definition,

$$\begin{aligned}
\frac{\Pi(K)}{\Pi(I)} &= \Pi(||K||)\Pi(||K|| - j_{|J|}) \cdots \Pi(||I|| + j_1) \\
&= (p - p^{p^{||K||}})(p - p^{p^{||K||-j_{|J|}}}) \cdots (p - p^{p^{||I||+j_1}}) \\
&\equiv p^{|J|} \bmod (p^{|J|-1+p^{||I||+j_1}}) \\
&\equiv p^{|J|} \bmod (p^{|K|+1}) \quad \text{since} \\
|J| - 1 + p^{||I||+j_1} &\geq |J| - 1 + ||I|| + 2 \\
&\geq |K| + 1.
\end{aligned}$$

Page 150, line 4: mod $(p^{1+||I||})$
Page 152, 4.4.3 (a) first line: $a_i \in \mathrm{Ext}^{1,2p^i-1}$
Page 153, Proof of 4.4.4, line 3: A1.3.12
Page 153, Proof of 4.4.4, last line: $\mathrm{Ext}_{E_*}(\mathbf{Z}/(p), \mathbf{Z}/(p))$
Page 156, line 2 of Proof of 4.4.14: $i < p - 1$
Page 156, line −4: ker δ_1
Page 157, line 6: $\beta_{p/i} = v_1^{1-i}\delta_1(v_2^p)$
Page 168, Figure 4.4.46: Move the label h_1 to the left
Page 169, line −1: $h_1 \cdot h_1 g = Pd_0$

Chapter 5

Page 170, line 6: say v_1^m. x' may

Page 174, 5.1.10, line 2: A1.2.11

Page 175, 5.1.16: $\Sigma^{\dim v_m} M_m^n$

Page 176, line 1: 5.1.16

Page 176, diagram in middle of page: longer arrow from M^n to N^{n+1}.

Page 176, line 3: $\mathrm{Ext}^s(M_m^n) \xrightarrow{v_m} \mathrm{Ext}^s(\Sigma^{-2p^m+2} M_m^n)$

Page 176, 5.1.18: largest integer

Page 178, line 6: $= \dfrac{-tv_2^{t-1}t_1^p|t_1}{v_1^2} - \cdots$

Page 180, line 12: 4.3.21

Page 180, line −6: $(-1)^n \dfrac{v_{n-1}^{p-1}t_1}{pv_1 \cdots v_{n-2}}$

Page 181, line −3: $d(t_1^3) = -3t_1|t_1^2 - 3t_1^2|t_1$

Page 184, section 2 line 2: 5.2.14 and 5.2.17

Page 185, 5.2.3: $\eta_R(v_1^{sp^i}) \equiv v_1^{sp^i} + sp^{i+1}v_1^{sp^i-1}t_1$

Page 185, second line after 5.2.3: $\delta\left(\dfrac{v_1^{sp^i}}{p^{i+1}}\right)$

Page 190, bottom line: $i \equiv 1 \bmod (n-1)$

Page 191, line 2 after 5.2.14: subject of 5.2.2

Page 194, line −3: $r \neq -1$ when $s = 1$.

Page 195, 5.3.7(b) line 3: $\mu_{2t-1} \in \pi_{8t+1}^S$

Page 196, 5.3.8: α_{4k} should be $\bar{\alpha}_{4k}$.

Page 199, 5.3.14: v_i should be v_1.

Page 199, line −2: 3.1.28

Page 215, line −14: γ's and δ's. Toda

Page 217, Theorem 5.6.5: For $p \geq 3$ the following relations hold in Ext^4 for $s, t > 0$.

(i) $\beta_s\beta_{tp^k/r} = 0$ for $k \geq 1$, $t \geq 2$ and $r < a_{2,k}$.

(ii) $\beta_s\beta_{tp^2/p,2} = \beta_{s+t(p^2-p)}\beta_{tp/p}$.

(iii) For $t, k \geq 2$,

$$\begin{aligned} \beta_2\beta_{tp^k/a_{2,k}} &= \beta_{s+(tp-1)(p^{k-1}-p)}\beta_{tp^2/a_{2,2}} \\ &= (t/2)\beta_{s+(tp-1)p^{k-1}-(2p-1)p}\beta_{2p^2/a_{2,2}}. \end{aligned}$$

Chapter 6

Page 220, 6.1.1, line 2: $\overline{M} \otimes_{BP_*} K(n)_*$

Page 223, line −1: $\Gamma' \square_{\Sigma(n)} K(n)_*$

Page 224, 6.1.12, line 2: $\Sigma(n) \otimes_{K(n)_*} B(n)_*$

Page 224, line 2 after 6.1.13: It is clear from 6.1.13

Page 226, line 3 of Section 2: $\Sigma(n)$-comodule M. $K(n)^*$ should be $K(n)_*$ twice.

Page 227, paragraph 2, line 2: is not defined over $\mathbf{F}_p$.

Page 227, paragraph 3, line 1: moment

Page 228, 6.2.4, line 3: $\mathrm{Ext}(\mathbf{F}_p, M)$

Page 228, 6.2.5, line 1: $S(n) \otimes \mathbf{F}_q$

Page 228, last line of proof of 6.2.5: $\omega^{-1}(t_kS^k)\omega = \omega^{-1}t_k\omega^{p^k}S^k = \omega^{p^k-1}t_kS^k$.

Page 229, 6.2.6, line 5: determinant
Page 229, first line above 6.2.7: $x \in \mathbf{Z}_p$ and $x \in \mathbf{Z}_2^\times$
Page 230, 3 lines above 6.2.11: has a power in $\mathbf{Z}[x]$
Page 230, 6.2.11, line 2: is the maximal rank of an elementary abelian p-subgroup
Page 231, last line of 6.3.1: $0 \leq j < n$.
Page 232, 6.3.4 (b): $b_{i,j} \in H^{2pd_{n,i}} E^0 S(n)$
Page 232, line 3 of proof of 6.3.5:

$$H^*(L(n)) \cong H^*(L(n,m)) \otimes E(h_{i,j} \colon i > m)$$

Page 233, 6.3.7: If $i > n-1$ and $i > m/2$
Page 235, mid page: $(t_i - t_i^{p^n})$. The displayed formula should read

$$m_{i,j} = \begin{cases} \sum_{k\geq 0} p^k t_{kn+j-i}^{p^i} & \text{for } i \leq j \\ \sum_{k\geq 1} p^k t_{kn+j-i}^{p^i} & \text{for } i > j \end{cases}$$

Page 235, 6.3.12: and, for $p = 2$, $U_n \in S(n)_*$
Page 239, line 9: $\xi \in H^2 S(2)$
Page 243, proof of 6.3.28: is isomorphic
Page 250, line 6: 8 and 10
Page 253, line -10: as is its proof
Page 253, 6.5.2, lines 2–3: $\tilde{E}^2(\mathbf{C}P^m)$ whose restriction to $\tilde{E}^2(\mathbf{C}P^1)$
Page 253, 6.5.3: $X(m)$
Page 253, bottom line: $\mathbf{C}P^{m-1} \to BU$
Page 254, line 1: $\mathbf{C}P^m \to X(m)$
Page 254, 6.5.4, replace CP by $\mathbf{C}P$ throughout.
Page 254, 6.5.4 line 2: $x_E \in \tilde{E}^2(\mathbf{C}P^m)$
Page 254, line -8: $X(m)_{(p)}$
Page 255, 6.5.6: If $i < n+2$ and $i < 2(p-1)(n+1)/p$ then

Chapter 7

Page 258, 7.1.2: $\Gamma(n) \to \Gamma(n+k+1) \ldots d_r : E_r^{s,t} \to E_r^{s+r,t-r+1}$.
Page 259, line 3: the precise definitions; see 5.1.10 and A1.2.11.
Page 259, line 4: $\Gamma(n+k+1)$
Page 260, line 1 of proof of 7.1.9: [The description of D_n^0 is incorrect; there is a weak injective of the form $A(n-1)[\lambda_{n+i} : i \geq 0]$ with λ_{n+i} congruent to $p^{-1}v_{n+i}$ modulo decomposables. Details will be in a forthcoming paper.]
Page 261, line 7: $1/p$ is divisible
Page 261, line 2 above 7.1.11: adjoining $\frac{v_n^p}{p^{p+1}} - \frac{v_1^{-1}v_{n+1}}{p} - \frac{v_1^{p^n-1}v_n}{p^2}$
Page 261, 7.1.11: [This formula is incorrect, but the method of 7.2.4 can be adapted to show that such elements exist.]
Page 261, 7.1.12, line 3: $C_n^2 \subset N^2$
Page 262, 7.1.13, line 1: $t < |v_{n+1}^p|$
Page 263:, 7.2.3, last two lines: *with equality holding only if M is a weak injective (7.1.6).*
Page 264, paragraph beginning on line 3: [Leave first sentence as is.] In a finite range of dimensions, the group $\mathrm{Ext}^0(M)$ is finite. For each i where $\mathrm{Ext}^0(M)$ does not map onto $\mathrm{Ext}^0(M_i'')$, $\mathrm{Ext}^0(M_{i+1}')$ is larger than $\mathrm{Ext}^0(M_i')$. This increase can occur only finitely many time since each $\mathrm{Ext}^0(M_i'')$ is a subgroup of the finite group $\mathrm{Ext}^0(M)$. Thus for i sufficiently large, $\mathrm{Ext}^0(M)$ must

map onto $\mathrm{Ext}^0(M_i'')$. We can make this argument for any range of dimensions, so in the limit we get $\mathrm{Ext}^0(M)$ mapping onto $\mathrm{Ext}^0(M'')$.

Page 264, line 11: annihilated by I.

Page 264, last two paragraphs of proof of 7.2.3: [Leave first sentence as is and replace the rest with the following] The A-module splitting $N \to N^0$ induces a comodule splitting $N \otimes \Gamma \to N^0 \otimes \Gamma$. Let $f : N \to N^0 \otimes \Gamma$ denote the composite of this map with the comdule structure map on N. Let $\tilde{N}$ and $\overline{N}$ denote the kernel and image of f, so we have a SES

$$0 \to \tilde{N} \to N \to \overline{N} \to 0$$

with $N^0 \subset \overline{N} \subset N^0 \otimes \overline{\Gamma}$. It follows that $\tilde{N}$ is more highly connected than N and that $\mathrm{Ext}^0(\overline{N})$ is a quotient of $\mathrm{Ext}^0(N)$. Let $g(M)$ denote the Poincaré series for M. Then

$$g(\overline{N}) \leq g(\mathrm{Ext}^0(\overline{N}))g(\overline{\Gamma}).$$

We can define a complete decreasing filtration on N by $F^{i+1}N = \widetilde{F^iN}$. Then we have

$$\begin{aligned} g(N) &= \sum_{i\geq 0} g(\overline{F^iN}) \\ &\leq \sum_{i\geq 0} g(\mathrm{Ext}^0(F^iN))g(\overline{\Gamma}) \qquad \text{since } \overline{F^iN} \subset \mathrm{Ext}^0(F^iN)\otimes\overline{\Gamma} \\ &= g(\mathrm{Ext}^0(N))g(\overline{\Gamma}) \end{aligned}$$

as claimed. Now suppose we have equality above, i.e., for each i

$$g(\overline{F^iN}) = g(\mathrm{Ext}^0(F^iN))g(\overline{\Gamma}).$$

Since $\overline{F^iN} \subset \mathrm{Ext}^0(F^iN)\otimes\overline{\Gamma}$, this means that $\overline{F^iN} = \mathrm{Ext}^0(F^iN)\otimes\overline{\Gamma}$, which is a weak injective. Then a standard filtration argument says that N is itself a weak injective as claimed. Finally a similar argument says that the weak injectivity of each subquotient of M above inplies that of M itself. ■

Page 264, proof of 7.2.4, line 1: From 7.1.2

Page 265, line 2 after proof of 7.2.4: we can take $C_{1,1}^2$ to be a suitable quotient of $C_{1,1}^1$.

Page 265, line −4: $\left\{\frac{v_3^{1+i}v_2^jv_1^k}{p} : i,j,k \geq 0\right\}$

Page 267, first displayed formula in proof of 7.2.6:

$$\psi\left(\frac{v_1^{ip}}{ip^{ip+1}} - \frac{v_1^{-i}v_2^i}{ip}\right) \equiv \sum_{0<j\leq i}\left(\begin{array}{c} i-1 \\ j-1 \end{array}\right)\left(\frac{v_1^{jp}}{jp^{jp+1}} - \frac{v_1^{-j}v_2^j}{jp}\right)(t_1^p - v_1^{p-1}t_1)^{i-j} \quad \text{modulo} \quad \tilde{C}_1^1$$

Page 269, line 2 of 7.3.6: $\bigcap_{i\geq p^2} \ker r_i$

Page 269, last line: $x = \sum_{0\leq i<p^2} x_i \otimes t_1^i$

Page 270, line −2: by 7.3.6

Page 272, 7.3.7: [Slanted font throughout]

Page 272, 7.3.7, line 3: $\mathrm{Ext}_{G(1,1)}(A(2), C_{1,1}^2 \otimes Y^{p^2-1})$

Page 272, 7.3.8, line 1: $\mathrm{Ext}_{G(1,1)}(A(2), C_{1,1}^2 \otimes Y^{p-1})$

Page 273, 7.3.9, line 3: $\dfrac{v_2^{pj}}{pv_1^i}$
Page 273, 7.3.9, last line: $E_2^{s,0} = b_{11}E_2^{s-2,0}$.
Page 275, first displayed formula:

$$h_{11}\langle h_{11},\, h_{11},\, h_{11},\, h_{11},\, b_{11}^3\rangle = \langle h_{11},\, h_{11},\, h_{11},\, h_{11},\, h_{11}\rangle b_{11}^3.$$

Page 275, second line after 7.3.13: $(-2i-2, -qi(p^2-1))$
Page 280, element in first column of third row of table: $\beta_{5/5-i,2}h_{11}$
Page 283, line 7 of Theorem 7.4.3: $C^{s,t} = \oplus_{i\geq 0}R^{2+s+2i,t+i(p^2-1)q}$
Page 286, 7.4.8: $\underline{2}\beta_1^2\beta_5$ is in 101-stem
Page 287, line −4: $\pi_{45}(S^0)$
Page 288, line 3 after 7.4.12: $\pm\alpha_1\eta_3 = \beta_4 b_{11} + \beta_{6/3}b_{10}$
Page 289, line −3 above §5. $\dfrac{v_3^2 t_2}{3v_1v_2}$
Page 291: 476 $h_{11}\gamma_2$
Page 291: 484 $h_{20}\gamma_2$
Page 292: 758 β_{16}
Page 292: 761 $b_{20}\beta_2\gamma_2$
Page 297, 7.5.5: 602 $\underline{\beta_{5/5}^2\beta_{5/4}}$
Page 303: Second generator of 893-stem is $\underline{4}\beta_1^3\beta_2\beta_{14}$. First generator of 952-stem is $\beta_1^6 x_{724}$.
Page 304: Generator of 955-stem is $\underline{2}\beta_1\beta_{19}$. First generator of 990-stem is $\beta_1^7 x_{724}$.

Appendix 1
Page 307, line −14. ... as remarked above.
Page 307, line −8: ...(which took us...
Page 309, line 16: our $\mathcal{P}^i$
Page 309, first two displayed lines of A1.1.1: left unit or source, ... right unit or target,
Page 309, line −6: right A-module map via
Page 310, diagram near top: [Vertical arrow labelled Δ should point up, not down.]
Page 310, line −7: is **primitive** if
Page 310, bottom line: [Replace first $\Gamma\otimes M\otimes N$ by $\Gamma\otimes M\otimes\Gamma\otimes N$.]
Page 313, In the first commutative diagram one of the maps from $\mathrm{Hom}_A(M,N)$ is ψ_N^*
Page 313, fourth line in A1.1.7: $f_2c = cf_2$
Page 313, first line in A1.1.8: Let $f : (A,\Gamma) \to (B,\Sigma)$ be a map of Hopf algebroids.
Page 319, third line in A1.2.1: $\theta(f) = (\Gamma\otimes f)\psi_M$
Page 320, second line in A1.2.4: $0 \to N \to R^0 \to R^1 \to \cdots$
Page 320, in A1.2.5: $\mathrm{Cotor}_\Gamma^0(M,R^0) \stackrel{\delta_0}{\to} \mathrm{Cotor}_\Gamma^0(M,R^1) \stackrel{\delta_1}{\to} \cdots$
Page 321, mid page: $\mu = (\epsilon\otimes M)\alpha$
Page 321, line −2: $\phi : M\otimes_A N \to M\square_\Gamma(\Gamma\otimes_A N)$
Page 322, A1.2.9(a), line 6: and the image of each map is a direct summand over A.
Page 322, line −2: $h_{i+1}d_i + d_{i-1}h_i = f_i - f_i'$
Page 323, A1.2.10, last line: and the image of each map is a direct summand over A.

Page 323, line -2 in the proof of A1.2.9: By A1.2.8 (a) it extends from M^{i+1} to P^{i+1}

Page 323, line 2 in A1.2.11: $D^s_\Gamma(M) = \Gamma \otimes_A \overline{\Gamma} \otimes_A M$, where $\overline{\Gamma} = \ker \varepsilon$

Page 324, line 2 in A1.2.13: $\mathrm{Cotor}_\Gamma(M_1 \otimes_A M_2, N_1 \otimes_A N_2)$

Page 325, A1.2.16 (a): If $I \subset A$ is invariant (A1.1.12)

Page 326, line -1: Then $\partial_2^{n+1,s,*}\partial_1^{n,s,*} + \partial_1^{n,s+1,*}\partial_2^{n,s,*} = 0$

Page 327, line 3: $B^{p,*} \to B^{p+1,*}$

Page 327, line 5: $F^p_I B = \displaystyle\bigoplus_{r\ge p}\bigoplus_q B^{r,q,*}$

Page 327, line 7: the functor $C_\Gamma(L, \cdot)$

Page 327, line 8: so $H^{s,*}F_{II}B = C^s_\Gamma(L, M)$

Page 327, line -2 above A1.3.4: The two SSs converge

Page 329, line 1 in A1.3.10: Let $\overline{\Gamma}$ be the unit coideal

Page 331, A1.3.14: Let M be a right Φ-comodule and a N a left Γ-comodule.

Page 332, line 5: $E_2 = \mathrm{Cotor}_\Phi(M, \mathrm{Cotor}_\Gamma(\Phi \otimes_D A, N))$

Page 332, line 2 in the proof of A1.3.16: $C^*_\Phi(M, C^*_\Gamma(\Phi \otimes_D A, N))$

Page 332, line 5 in the proof of A1.3.16: $m \otimes i_2(\phi_1) \otimes \cdots \otimes i_2(\phi_s) i_1\varepsilon(\phi) \otimes \gamma_{s+1} \otimes \cdots \gamma_{s+t} \otimes n$

Page 333, line 2: The argument in the proof of Theorem A1.3.14 showing that

Page 333, line 15: $\tilde{D}^{0,t}_\Sigma(N) = C^t_\Sigma(\Sigma, N)$

Page 335, line 3: $\mathrm{Ext}_\Sigma(K, N)$

Page 336, 10: $d(x)y - \overline{x}d(y)$

Page 339, last line: or the degrees

Page 341, line 2: $F^pH^{p+q}/F^{p+1}H^{p+q}$

Page 342, -12: Both displayed matric Massey products should be

$$\left\langle (d_{r+1}(\alpha_1 \quad \alpha_1)), \begin{pmatrix} \alpha_2 & 0 \\ d_{r+1}(\alpha_2) & \alpha_2 \end{pmatrix}, \begin{pmatrix} \alpha_3 \\ d_{r+1}(\alpha_3) \end{pmatrix} \right\rangle.$$

Page 351 (e), line 5: $\beta^\epsilon P^{a/2}P^{b/2}\ldots$

Appendix 2

Page 355, line -2: e.g., one can extract

Page 355, bottom line: defined over R; see Chapter 7 of Silverman [1].

Page 356, line 2: then $F(x, y)$ will converge

Page 356, line -7: $\sum_{i>0}$

Page 358, proof of A2.1.9, second line: such that $\phi(f(x)) =$

Page 358, A2.1.10(a): $L = \mathbf{Z}[x_1, x_2, \ldots]$

Page 358, A2.1.10(b), line 2: $i = p^k - 1$

Page 359, A2.1.11, line 2: $0 < i < n$

Page 360, line 4: $\sum_{n>1} m_{n-1}(x + y)^n$

Page 360, line 9: $x + y + aC_{n+1}(x, y)$

Page 360, add to A2.1.14: We call such a triple a **matched pair**.

Page 360, omit square at end of A2.1.15.

Page 361, line 9: and $c : LB \to LB$

Page 361, line (-4) of proof: $\mathrm{mog}(x) = \log f^{-1}(x)$.

Page 362, line 1: Every formal group law

Page 362, first line of proof of A2.1.20: defined by (b) for all

Page 362, A2.1.21: $\displaystyle\sum_{i=1}^{q}{}^F$

Page 363, top line: unless $(j+1)$ is
Page 363, line 6 after A2.1.22: $L \otimes \mathbf{Z}_{(p)}$
Page 364, line -5: (A2.1.14)
Page 366, line 6: (A2.1.9)
Page 366, line 2 after A2.1.27: $\sum_{i,j\geq 0}{}^F t_i c(t_j)^{p^i} = 1$
Page 368, line 4 of proof of A2.1.29: $a_i = a_{n-i}$
Page 369, line 3 after proof of A2.1.29: A2.1.12, which is
Page 370, line 3 after A2.2.1:

$$\lambda_3 = \frac{v_3}{p} + \cdots$$

Page 371, first line of proof of A2.2.5: A2.1.27(d)
Page 372, last 2 lines:

$$f([p]_F(x)) = [p]_G(f(x));$$

since $f(x)$ has leading term ux for u a unit in R and the result follows.
Page 373, line 4: $[p]_F(x) = u_{p-1}x^p$
Page 374, lines 6–7: (See Corollary 7.5 of Silverman [1].)
Page 374, first line above A2.2.10: we will specify a formal
Page 374, line 3 of A2.2.10: (A2.1.25)
Page 375, line 14: $g(f(x)) +_F h(f(x))$
Page 377, 2 lines above A2.2.17: except that S^n is p^i instead of p.
Page 379, line 2: By the definition of F_n (A2.2.10) and A2.2.14 we have

Appendix 3
Page 380, line 6: along with the differentials

Figure A3.1
[Redo $t-s$ label in each figure.]
Page 382, bottom: $h_1x = h_3n = c_1^2$
Pages 381–383: [η-extensions in Im J should be indicated, i.e. broken lines should connect $h_0^3h_4$ with Pc_0, h_0^2i with P^2c_0, $h_0^{10}h_5$ with P^3c_0, h_0^2i, $h_0^2P^2i$ with P^4c_0, and h_0^7Q' with P^5c_0, $h_0^2P^4i$ with P^6c_0.]
Page 383: [Double lines on differentials should be removed.]
Page 385, line -14: $\mathbf{Z}/(8)$ for $k=3$,

Table A3.3
π_*^S at $p=2$ $\quad ^a$
Tangora's name for generator of 8-stem is c_0.
ANSS name for generator of 9-stem is $\alpha_1\beta_2$.
Toda's name for first generator of 21-stem is σ^3.
Tangora's name for second generator of 21-stem is h_1g.
ANSS name for second generator of 21-stem is $\alpha_1\beta_4$.
ANSS name for second generator of 23-stem is $x_{23} = \langle\alpha_{2/2}, \alpha_1^3, \beta_{4/3}\rangle$.
Tangora's name for third generator of 34-stem is e_0^2.
Tangora's name for second generator of 35-stem is $h_1e_0^2$.
Tangora's name for second generator of 37-stem is x.
Tangora's name for second generator of 38-stem is h_1x.
Tangora's name for fifth generator of 39-stem is u.

Tangora's name for third generator of 41-stem is z.

Table A3.4

3-primary Stable Homotopy Excluding Im J [a]

$\alpha_1\beta_1\beta_2$ in 39-stem.

First generator of 85-stem is $\langle\alpha_1,\alpha_1,\beta_2^3\rangle = \beta_1\mu$.

Generator of 90-stem is β_6.

Table A3.5

$\beta_1^4\beta_4$ in 334-stem, not 341-stem.

Generator of 341-stem should be $\alpha_1\beta_1^4\beta_4$.

Generator of 411-stem should be $\alpha_1 x_{404} = \beta_{5/4}\underline{2}\beta_1^5$.

$\alpha_1\gamma_2$ is in the 444-stem, not the 443-stem.

Generator of 514-stem is $\beta_1^2\beta_{10/5}$.

β_{12} is in 566-stem, not 565-stem.

$\beta_2\gamma_2$ is in 523-stem, not 574-stem.

$\underline{4}\beta_1^{16}$ is in 639-stem, not 636-stem.

$\alpha_1 x_{636}$ is second generator of 643-stem.

Generator of 689-stem is $\underline{3}\beta_1^5\underline{5}\gamma_2$.

Second generator in 642-stem is $\underline{2}\beta_1^5\gamma_2$.

There is a second generator in the 643-stem, $\alpha_1 x_{636}$.

$\underline{2}\beta_2\beta_{14}$ in 763-stem.

Generator of 812-stem is $\beta_3\beta_{15/5}$.

840 is mislabeled as 810.

Second generator of 893-stem is $\underline{4}\beta_1^3\beta_2\beta_{14}$.

Generator of 934-stem is $\beta_{20/3}$.

Second generator of 940-stem is $\beta_1\beta_{19}$.

First generator of 952-stem is $\beta_1^6 x_{724}$.

$\underline{3}\beta_1^{13}\gamma_2$ in 954-stem.

Generator of 955-stem is $\underline{2}\beta_1\beta_{19}$.

Second generator of 978-stem is $\beta_1^2\beta_{19}$.

Generator of 989-stem is $\beta_1^6 x_{761}$.

First generator of 990-stem is $\beta_1^7 x_{724}$.

Second generator of 990-stem is $\beta_1 x_{952}$.

Figure A3.6

Line -2: For $n > k+2$ the group is isomorphic to the one for $n = k+2$.

References

Page 395, Adams [6] and [7]: Springer-Verlag, Berlin and New York.

Page 395, Adams [9]: Princeton University Press.

Page 395: Adams et al [18] Topology 24(1985), 435–460.

Page 396: Bahri, A. P., and Mahowald, M. E.

Page 396, Bousfield and Kan [1]: Springer-Verlag, Berlin and New York.

Page 397, Bruner [1]: *Lecture Notes in Math.* **1176** (1986) Springer-Verlag, Berlin and New York.

Page 397, Bruner [2] should not have coauthors. Page numbers are 271–276.

Page 397: R. L. Cohen [2]: *Mem. Amer. Math. Soc.*

Page 397, Conner and Floyd [1] and [4]: Springer-Verlag, Berlin and New York.

Page 398, Fröhlich: Springer-Verlag, Berlin and New York.

Page 398, Giambalvo and Pengelley [1]: The homology of $MSpin$

Page 399, add: T. Kambe, H. Matsunaga and H. Toda [1] A note on stunted lens spaces, *J. Math. Kyoto Univ.* **5**(1966), 143–149.

Page 400, Liulevicius [2]: *Mem. Amer. Math. Soc.*

Page 400, Liulevicius [3]: Springer-Verlag, Berlin and New York.

Page 400, Mahowald [10] and [11]: Springer-Verlag, Berlin and New York.

Page 401, add: Makinen, J. Boundary Formulae for reduced powers in the Adams spectral sequence, *Ann. Acad. Sci .Fennicae Ser. A* (1973), No. 562.

Page 401, Massey [1]: *Graduate Texts in Matahematics* **56** (1977), Springer-Verlag, Berlin and New York.

Page 401, May [5]: Springer-Verlag, Berlin and New York.

Page 401, Miller and Wilson [3]: On Novikov's Ext^1 modulo an invariant prime ideal, *Topology* **15** (1976), 131-141.

Page 401, Miller and Ravenel [5]: Novikov

Page 401, Milnor [1] and [5]: Princeton University Press.

Page 402, Morava [2]: Springer-Verlag, Berlin and New York.

Page 402, Mumford [1]: *Ann. Math. studies* **59**

Page 402, add to Nakamura: [3] Some differentials in the mod 3 Adams spectral sequence, *Bull. Sci. Engrg. Div. Univ. Ryukyus (Math. Nat. Sci.)*, **19**(1975), 1–26.

Page 403, Oka [7]: *Contemp. Math.* **19**(1983), 282-308.

Page 403: Quillen [3] is out of place; it should follow Quillen [2].

Page 404, Serre [1]: "Local fields." *Graduate Texts in Math.* **67** (1979) Springer-Verlag, Berlin and New York.

Page 404, add: Silverman, J. H. [1] "The Arithmetic of Elliptic Curves." *Graduate Texts in Math.* **106** (1986), Springer-Verlag, Berlin and New York.

Page 404, Steenrod and Epstein [1]: Princeton University Press.

Page 404, bottom, works of Tangora are out of place; they should follow Switzer on the next page. Add to these:

[6] Some homotopy groups mod 3. *Reunion sobre teoria de homotopia*, Northwestern Univ. 1974, Soc. Mat. Mexicana (1975) 227–245.

Page 404, Tangora [2] and [3]: Springer-Verlag, Berlin and New York.

Page 405, Toda [1], On spectra realizing exterior parts of the Steenrod algebra.

Page 405, Toda [5]: Princeton University Press.

Page 405, Welcher [2]: K(n)-homology, ... *Indiana J. Math.*

Page 405, Wellington [5], remove repetition.

Page 405, Wilson [1]: cohomology

Page 406, line 1: Würgler, U.

Index

Page 410: Landweber-Novikov theorem

Page 411: Poincaré series