PRINCETON UNIVERSITY

Special Algebraic Topology Seminar

What is the telescope conjecture?

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Two developments in the early 70s

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It is related to height n formal group laws, and $K(n)_*(K(n))$ is related to the Morava stabilizer group $\mathbb{G}_n$. It is a p-adic Lie group and the automorphism group of a height n formal group law over a suitable field of characteristic p.

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for $1 \le n \le 3$ and $p \ge 2n+1$. We know that $K(n)_*V(n-1) \neq 0$ and w_n is a $K(n)$-equivalence. These lead to the construction of the v_n-periodic families aka Greek letter elements

$\alpha_t \in \pi_{t\lvert v_1\rvert-1}\mathbb{S}$	for $p \ge 3$
$\beta_t \in \pi_{t\lvert v_2\rvert-2p}\mathbb{S}$	for $p \ge 5$
$\gamma_t \in \pi_{t\lvert v_3\rvert-2p^2-2p+1}\mathbb{S}$	for $p \ge 7$

Is there more?

The Adams-Novikov spectral sequence

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The chromatic filtration

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Let Sp denote the category of spectra. Given a homology theory E_*, Bousfield constructed an endofunctor $L_E : \mathrm{Sp} \to \mathrm{Sp}$ whose image category $L_E\mathrm{Sp}$ is stable homotopy as seen through the eyes of E-theory.

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We are interested in the case $E = K(n)$. $L_{K(n)}\mathrm{Sp}$ is much easier to deal with than Sp itself. For example, we can compute $\pi_* L_{K(2)} V(1)$, but have no hope of computing $\pi_* V(1)$.

The Hopkins-Smith periodicity theorem

Recall the cofiber sequence

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$$V(n-1) \xrightarrow{w_n} \Sigma^{-|v_n|} V(n-1) \xrightarrow{w_n} \Sigma^{-2|v_n|} V(n-1) \xrightarrow{w_n} \dots$$

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Again the telescope conjecture equates the geometrically appealing telescope $w^{-1}X$ with the computationally accessible Bousfield localization $L_{K(n)}X$.

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San Francisco earthquake of October 17, 1989

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