Math 162: Calculus IIA

Second Midterm Exam

November 15, 2022

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Your University email ______________________________

Indicate your instructor with a check in the box:

Sefika Kuzgun	MW 9:00 - 10:15 AM	
Doug Ravenel	MWF 10:25 - 11:40 AM	
Josh Sumpter	TR 9:40 - 10:55 AM	
Carissa Slone	TR 2:00 - 3:15 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: ______________________________

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. IF YOU HAVE YOUR PHONE WITH YOU, YOU MUST TURN IT IN TO A PROCTOR BEFORE STARTING THE EXAM. FAILURE TO DO SO WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Put your answers in the space provided at the bottom of each page or half page.**
- **You are responsible for checking that this exam has all 15 pages.**

Integration by parts formula:

$$\int u\,dv = uv - \int v\,du$$

Trigonometric identities:

$$\cos^2(x) + \sin^2(x) = 1 \qquad \sec^2(x) - \tan^2(x) = 1 \qquad \sin(2x) = 2\sin(x)\cos(x)$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2} \qquad \sin^2(x) = \frac{1-\cos(2x)}{2}$$

Derivatives of trig functions.

$$\frac{d\sin x}{dx} = \cos x \qquad \frac{d\tan x}{dx} = \sec^2 x \qquad \frac{d\sec x}{dx} = \sec x\tan x$$

$$\frac{d\cos x}{dx} = -\sin x \qquad \frac{d\cot x}{dx} = -\csc^2 x \qquad \frac{d\csc x}{dx} = -\csc x\cot x$$

Trigonometric substitution tricks for odd powers of secant and even powers of tangent:

$$u = \sec(\theta) + \tan(\theta) \qquad \sec(\theta)d\theta = \frac{du}{u}$$

$$\sec(\theta) = \frac{u^2+1}{2u} \qquad \tan(\theta) = \frac{u^2-1}{2u}$$

More formulas for your enjoyment

Polar coordinates

$$r = \sqrt{x^2 + y^2} \qquad \theta = \arctan(y/x) \qquad \text{for } x > 0$$
$$\pi + \arctan(y/x) \text{for } x < 0$$
$$\pi/2 \text{for } x = 0 \text{ and } y > 0$$
$$3\pi/2 \text{for } x = 0 \text{ and } y < 0$$
$$\text{undefined} \text{for } (x, y) = (0, 0)$$
$$x = r\cos\theta \qquad y = r\sin\theta$$

Changing θ by any multiple of 2π does not change the location of the point. Changing the sign of r is equivalent to adding π to θ, which is the same as moving the point to one in the opposite direction and the same distance from the origin.

Area in polar coordinates for $r = f(\theta)$ with $\alpha \leq \theta \leq \beta$:

$$A = \int_\alpha^\beta \frac{r^2}{2}\, d\theta$$

Arc length formulas

- Rectangular coordinates, $y = f(x)$ with $a \leq x \leq b$:

$$S = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

- Polar coordinates, $r = f(\theta)$ with $\alpha \leq \theta \leq \beta$:

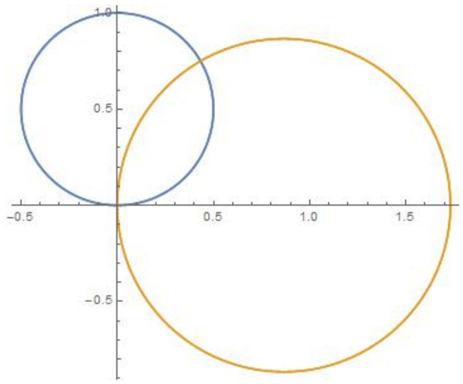
$$S = \int_\alpha^\beta \sqrt{r^2 + f'(\theta)^2}\, d\theta$$

- Parametric equations, $x = x(t)$ and $y = y(t)$ with $a \leq t \leq b$:

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

1. (20 points)

(a) Find the area of the region both inside the circle $r = \sin\theta$ and outside the circle $r = \sqrt{3}\cos\theta$ (both equations are in polar coordinates). The two circles are shown below. THEY INTERSECT AT THE ORIGIN AND THE POLAR POINT $(\theta, r) = (\pi/3, \sqrt{3}/2)$.



ANSWER:

(b) Compute the equation (in Cartesian coordinates x, y) of the tangent line to the circle $r = \sin\theta$ at the points where it intersects the circle $r = \sqrt{3}\cos\theta$

ANSWER:

2. (20 points)

(a) (10 points) Find

$$\int_a^\infty \frac{dx}{(x+4)^{3/2}} \qquad \text{for } a \geq 0.$$

ANSWER:

(b) (10 points) Find

$$\int_4^{\infty} e^{-x/4} dx$$

ANSWER:

3. (20 points)

Determine if the following sequences are convergent or divergent and explain why. If it is convergent, give its limit.

(a) (10 points)

$$\left\{ \frac{3n\sin(n)}{2n^3+5} \;\middle|\; n \geq 0 \right\}.$$

ANSWER:

(b) (10 points)

$$\left\{ \frac{n}{\ln(n)} \;\middle|\; n \geq 2 \right\}$$

ANSWER:

4. (20 points)

(a) Compute the area of surface of revolution obtained by rotating the following curve around the x-axis:

$$y = \sqrt{1 + e^x} \quad 0 \leq x \leq 1$$

ANSWER:

(b) Compute the area of surface of revolution obtained by rotating the following curve around the y-axis:

$$y = \frac{x^2}{2} \quad 0 \leq x \leq 1$$

ANSWER:

5. (20 points)

Find the arc length of the curve described by the parametric equations

$$x = \cos(t^2), \quad y = \sin(t^2)$$

between the points with Cartesian coordinates $(1, 0)$ and $(-1, 0)$.

ANSWER:

Scratch paper

Scratch paper

Scratch paper