Math 162 final exam
December 12, 2021

Handy dandy formulas

Integration by parts formula:

$$\int u\,dv = uv - \int v\,du$$

Trigonometric identities:

$$\cos^2\theta + \sin^2\theta = 1 \qquad \sec^2\theta - \tan^2\theta = 1$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Derivatives of trig functions.

$$\frac{d\sin x}{dx} = \cos x \qquad \frac{d\tan x}{dx} = \sec^2 x \qquad \frac{d\sec x}{dx} = \sec x\tan x$$

$$\frac{d\cos x}{dx} = -\sin x \qquad \frac{d\cot x}{dx} = -\csc^2 x \qquad \frac{d\csc x}{dx} = -\csc x\cot x$$

Trigonometric substitution for integrals of the form

$$\int \tan^m x\sec^n x\,dx \qquad \text{with } n > 0,$$

known in Doug's section as *the rabbit trick.*

$$u = \sec x + \tan x \qquad \sec x\,dx = \frac{du}{u}$$

$$\sec x = \frac{u^2+1}{2u} \qquad \tan x = \frac{u^2-1}{2u}$$

Area of surface of revolution in rectangular coordinates, $y = f(x)$ with $a \leq x \leq b$

- about the x-axis: $S = 2\pi\displaystyle\int_a^b f(x)\sqrt{1+f'(x)^2}\,dx$
- about the y-axis: $S = 2\pi\displaystyle\int_a^b x\sqrt{1+f'(x)^2}\,dx$

More formulas for your enjoyment

Polar coordinates

$$r = \sqrt{x^2 + y^2} \qquad \theta = \arctan(y/x) \quad \text{for } x > 0$$

$\pi + \arctan(y/x)$for $x < 0$

$$\pi/2\text{for } x = 0 \text{ and } y > 0$$

$$3\pi/2\text{for } x = 0 \text{ and } y < 0$$

$$\text{undefinedfor } (x, y) = (0, 0)$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

Changing θ by any multiple of 2π does not change the location of the point. Changing the sign of r is equivalent to adding π to θ, which is the same as moving the point to one in the opposite direction and the same distance from the origin.

Area in polar coordinates for $r = f(\theta)$ with $\alpha \le \theta \le \beta$:

$$A = \int_\alpha^\beta \frac{r^2}{2}\, d\theta$$

Arc length formulas

- Rectangular coordinates, $y = f(x)$ with $a \le x \le b$:

$$S = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

- Polar coordinates, $r = f(\theta)$ with $\alpha \le \theta \le \beta$:

$$S = \int_\alpha^\beta \sqrt{r^2 + f'(\theta)^2}\, d\theta$$

- Parametric equations, $x = x(t)$ and $y = y(t)$ with $a \le t \le b$:

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Infinite series formulas

The Maclaurin series for $f(x)$ is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n.$$

The Taylor series for $f(x)$ at a is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n.$$

The mth Taylor polynomial is

$$T_m(x) = \sum_{n=0}^{m} \frac{f^{(n)}(a)}{n!} (x-a)^n,$$

and the mth Taylor remainder is

$$R_m(x) = f(x) - T_m(x)$$

Taylor's inequality says that if $|f^{(n+1)}(x)| \leq M$ for suitable x, then

$$|R_m(x)| \leq \frac{(x-a)^{n+1} M}{(n+1)!}.$$