Restricted convolution inequalities, multilinear operators and applications. Clarification and errata.

Dan-Andrei Geba, Allan Greenleaf, Alex Iosevich,
Eyvindur Palsson and Eric Sawyer

Pagination and numbering refer to the published version, *Math. Res. Let.* **20** (2013), *no.* 4, 675-694.

1. Clarification

It would have been helpful to explain the exponents in the main theorem, Thm. 1.3, by means of scaling. Observe how both sides of a possible more general estimate,

$$(1.4') \qquad ||(F * G)|_H||_{L^r(H)} \leq ||F||_{\Lambda^H_{s,p}(\mathbb{R}^n)} \cdot ||G||_{\Lambda^H_{t,q}(\mathbb{R}^n)},$$

transform under dilations:

(i) Dilating by $0 < \delta < \infty$ in the H directions and not in the $H^\perp$ directions, the LHS of (1.4') scales by $\delta^{-\frac{k}{r'}}$, while the RHS scales by $\delta^{-\frac{k}{p}} \cdot \delta^{-\frac{k}{q}}$, so that (1.4') holding uniformly in δ implies that

$$\frac{1}{r'} = \frac{1}{p} + \frac{1}{q},$$

i.e., $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1$, as in Thm. 1.3.

(ii) Similarly, dilating by $0 < \epsilon < \infty$ in the $H^\perp$ directions and not in the H directions scales the LHS by $\epsilon^{-(n-k)}$ and the RHS by $\epsilon^{-\frac{n-k}{s}} \cdot \epsilon^{-\frac{n-k}{t}}$, so that (1.4') holding uniformly in ϵ implies that

$$1 = \frac{1}{s} + \frac{1}{t},$$

i.e., s, t are dual exponents. Thm. 1.3 only covers the case $s = t = 2$.

The first three authors were supported by NSF grants DMS-0747656, DMS-0853892 and DMS-1045404, resp.
2010 *Mathematics Subject Classification:* Primary 44A35; Secondary 42B25, 35B45 .

2. Corrections

There are also a number of typographical errors which might cause confusion.

1. p. 678, in (1.10) in Cor. 1.7, the spaces are incorrect due to a transcription error. p and q should have been $\frac{p}{2}$, $\frac{q}{2}$, resp., so that the inequality should have been

$$(1.10) \quad \left|\left|\widehat{F}|_H\right|\right|_{L^r(H)} \lesssim \left|\left|F \circ \rho_H\right|\right|^{\frac{1}{2}}_{L_u^{\frac{p}{2}} L_v^1} \cdot \left|\left|F \circ \rho_H\right|\right|^{\frac{1}{2}}_{L_u^{\frac{q}{2}} L_v^1}, \quad p, q, r \geq 2, \ \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1.$$

The estimate (1.11) is correct as stated, but (replacing p by $p/2$), is perhaps more elegantly expressed as

$$(1.11) \qquad \left|\left|\widehat{F}|_H\right|\right|_{L^{p'}(H)} \lesssim \left|\left|F \circ \rho_H\right|\right|_{L_u^p L_v^1}, \quad 1 \leq p \leq 2.$$

We thank Mike Christ for pointing these out.

2. p. 680, proof of Thm. 1.3, just above §§2.2, should read

> Interpolation then gives (1.4) for $q = 2$, $\frac{1}{p} + \frac{1}{r} = \frac{1}{2}, p, r \geq 2 \ldots$ also holds for $p = 2$, $\frac{1}{q} + \frac{1}{r} = \frac{1}{2}$, $q, r \geq 2 \ldots$

3. p. 684, proof of Cor. 3.5: should be

> ... (3.2) holds with $\gamma = \frac{md-1}{2} - \frac{(m-1)d}{2} = \frac{d-1}{2}$, using ...

There is also an example which is incorrect and should be removed:

4. p. 685, Cor. 3.6: For the measure on the product of spheres, B_ν is just the pointwise product of the spherical averages on $\mathbb{R}^d$ of each of the f_j. One can't beat simply applying Strichartz' $L^{\frac{d+1}{d}} \to L^{d+1}$ estimate for the spherical mean operator for each of these, followed by Hölder.

Department of Mathematics, University of Rochester, Rochester, NY 14627
E-mail address: dangeba@math.rochester.edu
E-mail address: allan@math.rochester.edu
E-mail address: iosevich@math.rochester.edu

Department of Mathematics and Statistics, Williams College, Williamstown, MA 01267
E-mail address: eap2@williams.edu

Department of Mathematics and Statistics, McMaster University, Hamilton, ON L8S 4K1
E-mail address: sawyer@mcmaster.ca