You have seen Dans lecture on complex #s, and if necessary practiced the problems in my lec20.

<u>Thm</u>: Polynomial $P(\lambda)$ of degree n with real coefficients - then it has n roots where each root is counted with its multiplicity.

$$P(x) = \overset{\text{account for sign}}{\pm}(x-\lambda_1)^{m_1} \cdots\cdots (x-\lambda_k)^{m_k}$$

$m_1 \to$ positive integer

$$\sum_{i=1}^{k} m_i = n$$

If λ is a complex root, then $\bar{\lambda}$ is also a root. $(\lambda = a+ib,\ \bar{\lambda} = a-ib)$.

$$\mathbb{C}^2 = \{(\underset{a_1+ib_1}{z_1}, \underset{a_2+ib_2}{z_2}) : z_1, z_2 \in \mathbb{C}\}$$

$$= \text{span}\{(1,0),(0,1)\}$$ (if you allow complex scalars)

$$(z_1, z_2) = z_1(1,0) + z_2(0,1)$$

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$ (a pair of DEs)

$$Y' = AY$$

$$\begin{bmatrix} 1 & 1 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$\underline{\lambda = 2}$ $m_2 = 2$

$$B^{\#} = \left[\begin{array}{ccc|c} 3 & 12 & -6 & 0 \\ -3 & -12 & 6 & 0 \\ 3 & -12 & 6 & 0 \end{array}\right]$$

$$(A - \lambda I) \cdot v = \vec{0}$$

$$\begin{bmatrix} 5 & 12 & -6 \\ -3 & -10 & 6 \\ -3 & -12 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\sim \left[\begin{array}{ccc|c} 3 & 12 & -6 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]_{3 \times 3} = n$$

$\text{rank}(B^{\#}) = 1$

$\text{nullity}(B) = 3-1 = 2 = \dim(\ker(B))$

$= \dim(\text{solutionspace})$

$3v_1 + 12v_2 - 6v_3 = 0$

$v_2 = s, \quad v_3 = t$

$v_1 = -4s + 2t$

$S = \{(-4s+2t, s, t)\}$

$E_4 = \text{span}\{(-4,1,0), (2,0,1)\}$

(Any vector in this subspace is an eigenvector)

$= \text{span}\left\{ \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right\}$

$\lambda = -1$ case $\begin{bmatrix} 6 & 12 & -6 \\ -3 & -9 & 6 \\ -3 & -12 & 9 \end{bmatrix}$

$B \sim \begin{bmatrix} 1 & 2 & -1 \\ -1 & -3 & 2 \\ -1 & -4 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ rank(2) $\quad$ null $= 3-2 = 1$

$v_1 + 2v_2 - v_3 = 0$

$v_2 - v_3 = 0$

$v_3 = t \Rightarrow v_2 = t$

$\Rightarrow v_1 = -t$

$S = \text{span}\{(-1, 1, 1)\} = E_{-1}$

distinct $\lambda_1 = -1$ $\lambda = 4$

$M = \{ (-1, 1, 1), \underbrace{(-4, 1, 0), (2, 0, 1)}_{\text{linearly independent.}} \}$

$\Rightarrow$ all vectors in M are linearly indep.

$$\frac{\cancel{-6}\, 2}{\cancel{-3}(1+i)} \times \frac{(1-i)}{(1-i)} = \frac{2(1-i)}{1+1}$$

$\lambda \cdot \bar{\lambda} = |\lambda|^2$

$$= \frac{\cancel{2}(1-i)}{\cancel{2}} = 1 - i$$

Image of $\mathbb{R}^n$ under A

$$\{Ax : x \in \mathbb{R}^n\}$$

$$A = [v_1 \cdots\cdots v_n]$$

$$= \{x_1 v_1 + \cdots + x_n v_n\}$$

$$= \operatorname{span}\{v_1 \cdots v_n\} = \operatorname{colspan}(A)$$

$$\overbrace{(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4)}$$

$$(a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3)$$

$$(2a_2 + 6a_3 x + 12 a_4 x^2) = 0$$

$$b_1 + b_2 x + b_3 x^2 \in \text{range}$$

$$= \operatorname{span}\{1, x, x^2\}.$$