Lec20 Complex #s

Ex: $x^2 + 2x + 17 = 0$ Find roots of this polynomial.

$x = \frac{-2 \pm \sqrt{4 - 4 \cdot 17}}{2} = -1 \pm \frac{1}{2}\sqrt{-4 \cdot 16} = -1 \pm \frac{1}{2} i 8$

$= -1 \pm 4i$

Ex: $7 + 4i + 3 - 8i =$

Ex: Find $\overline{3 + 2i}$

Ex: Multiplication

$(2+3i)(7-i)$

Ex: $\overline{(2+3i)(7-i)} =$

Ex: Magnitude of the number $2+3i$

$|z|^2 = z\bar{z}$ $\quad |2+3i|^2 = 4 + 9 = 25$

$= (2+3i)(2-3i) = 4 - 6i + 6i - 9i^2$

Division: $\dfrac{(2+3i)}{7} = \dfrac{2}{7} + \dfrac{3}{7}i$

(Division by real #s OK).

So $\dfrac{z_1}{z_2} := \dfrac{z_1\,\overline{z_2}}{z_2\,\overline{z_2}} = \dfrac{z_1\overline{z_2}}{|z_2|^2}$

Find $\dfrac{1+7i}{2-3i}$

Complex exponentiation.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$$

$$e^{ix} = 1 + \frac{ix}{1!} + \frac{(i^2x^2)}{2!} + \cdots \qquad x \in \mathbb{R}$$

$i^{2k} = (-1)^k \qquad i^{2k+1} = (-1)^k i$

$$= 1 + \frac{ix}{1!} - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} \cdots$$

$$= \underbrace{\left(1 - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots\right)}_{\cos x} + i \underbrace{\left(x - \frac{x^3}{3!} + \cdots\right)}_{\sin x}$$

$$e^{a+ib} = e^a e^{ib} = e^a(\cos b + i \sin b)$$

Note $|e^z| = e^a \underbrace{|e^{ib}|}_{\sqrt{\cos^2 b + \sin^2 b}} = 1$

So $e^a = e^{Re(z)}$ magnitude of e^z.

Ex: e^{2+3i}, $e^{-4+\frac{\pi}{4}i}$

Polynomials with real coefficients

Ex

$P(x) = x^4 + 2x^2 + 1 = (x^2+1)^2 \quad [(x+i)(x-i)]^2$

2 roots $\quad i$ with multiplicity 2

$\quad -i$ with multiplicity 2

Complete factorization of $P(x) = a_n x^n + \cdots a_1 x + a_0$

$P(x) = (x-\lambda_1)^{m_1} \cdots (x-\lambda_k)^{m_k}$

m_i = multiplicity of root λ_i

$$\sum_{i=1}^{k} m_i = n$$

Complex roots appear in pairs: if λ is a root so is $\bar{\lambda}$ and they have the same multiplicity