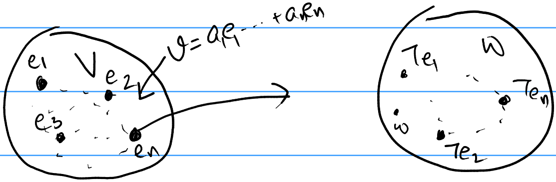
$V = \mathbb{R}^n \qquad \dim(V) = n.$ Take a basis in V.

$e_1 = (1, 0, \cdots 0) \quad e_2\,(0, 1, \cdots) \quad \cdots$

$\tilde{e}_1 = (1, 1, \cdots 0) \quad \tilde{e}_2 = (0, 1, \cdots) \quad \cdots$



1) $T(y_1 + y_2) = Ty_1 + Ty_2 \quad \forall\, y_1 \,\&\, y_2$

2) $T(\lambda y) = \lambda T y \quad \forall\, y$

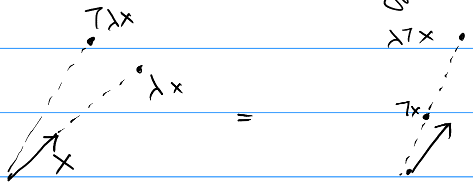
$C^2 = \{ f : \mathbb{R} \to \mathbb{R} \mid f, f', f'' \text{ continuous} \}$

$C^0 = \{ f : \mathbb{R} \to \mathbb{R} \mid f \text{ continuous} \}$

$C(\mathbb{R}) = \{ \text{set of continuous functions} \}.$

$$\underset{2\times2}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{2\times1}{\begin{bmatrix} \underbrace{x_1 \begin{bmatrix} a \\ c \end{bmatrix} + x_2 \begin{bmatrix} b \\ d \end{bmatrix}} \end{bmatrix}}$$

linear comb of the columns of a matrix where x_1, x_2 are the coefficients.



$$y'' + y(t) = 0 \qquad y = A\cos t + B\sin t$$

$\ker(T) = \{ v : Tv = \vec{0} \in W \}$ $\quad T: V \to W$

$\underbrace{\operatorname{ran}(T)} = \{ Tv : v \in V \}$ (image of T)

ker and ran are subspaces of V and W.

$T: A \longrightarrow A^{-1}$ (is this linear)

V : set of invertible matrices.

$\det(A) \neq 0$

Is V a vector space?

$(A+B)^{-1}$ exist if $\det(A) \neq 0$ $\det(B) \neq 0$.

$\det(A+B) \neq 0$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

The domain is not a vector space.

$V = M_{2\times2}$ $\quad W = M_{3\times3}$

$\downarrow$

$$a\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_{e_1} + b\underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{e_2} \cdots\ \cdots\ d\underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}_{e_4}$$

$$\underbrace{\begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}}_{D}{}_{9\times 4} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ x_9 \end{bmatrix}_{9\times 1}$$

$$x_1\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \cdots$$