Google: Arjun Krishnan (Rochester)
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(Do we need to upload individual questions for the midterm?)

From the 1st row: $x_1 + x_2 + 2x_3 + 3x_4 = 0$

$x_2 - 7x_3 - 7x_4 = 0$

$\Rightarrow x_2 = 7t_3 + 7t_4$

$$\begin{pmatrix} -9t_3 \\ 7t_3 \\ t_3 \\ 0 \end{pmatrix} + \begin{pmatrix} -10t_4 \\ 7t_4 \\ 0 \\ t_4 \end{pmatrix}$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 a_1 + x_2 a_2 + x_3 a_3$$

$$= b$$

UNIQUE sol. if rank(A) = n,
the # of column of A.

$$\begin{bmatrix} & \\ & \end{bmatrix}_{4\times 3}$$

4 rows and 3 columns.
3 unknowns.
rank(A) $\leq$ 3

If $b \in$ colspace(A) Then if rank(A) = 3
then $Ax = b$ has a unique soln.

Question: Given

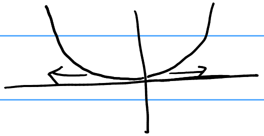
$f(x) = x^3$ $g(x) = |x|^3$

Determine if f and g are lin indep.

Can you use the wronskian? How
do you differentiate g.

$$g(x) = \begin{cases} x^3 & x > 0 \\ -x^3 & x \leq 0 \end{cases}$$

$$g'(x) = \begin{cases} 3x^2 & x > 0 \\ -3x^2 & x \leq 0 \end{cases}$$



$g'(x)$ is continuous everywhere

$$\lim_{x \to 0^+} g'(x) = \lim_{x \downarrow 0^+} 3x^2 = 0$$

$$\lim_{x \uparrow 0^-} g'(x) = \lim_{x \uparrow 0^-} -3x^2 = 0$$

So you can use the Wronskian.

$$W(x) = \begin{cases} \det \begin{bmatrix} x^3 & x^3 \\ 3x^2 & 3x^2 \end{bmatrix} & x > 0 \\ \det \begin{bmatrix} x^3 & -x^3 \\ 3x^2 & -3x^2 \end{bmatrix} & x < 0 \end{cases}$$

$$W(x) = \begin{cases} 0 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

So the WRONSKIAN TEST does not say anything.

$$f(x) = k\,g(x) \quad \text{(Suppose dep.)}$$

$$x^3 = k|x|^3 \qquad \forall\, x$$

$$\left.\begin{aligned} 1 &= k\,1 \\ -1 &= k\,1 \end{aligned}\right] \quad \begin{aligned} &\text{contradiction} \\ &x = 1 \\ &x = -1 \end{aligned}$$

$\Rightarrow$ f and g are linearly independent.