- Basis and dimension
- Row and column

- Rank Nullity theorem

$S = \{ x \in \mathbb{R}^3 \mid \underbrace{x_1 + 2x_2 - x_3}_{} = 0 \}$

A symmetric has equal entries on either side of the main diagonal.

$C^n(I)$ $I = [a,b]$ consists of the set of functions f that have n continuous derivatives.

If I find vectors $\{v_1, \dots v_{M+1}\} \in C^n$

Then $\underbrace{\text{span}\{v_1, \dots v_{M+1}\}}_{\dim = M+1} \subseteq C^n$

$\dim = M+1 \ngtr \dim(C^n) = M$

Thats a contradiction since this is a subspace of C^n

The det of an upper Δ matrix is the product of non zero entries.

$A \sim \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$L = L_1 \cdots L_k$ and each L_i is invertible

is L invertible? $\det(L_i) \neq 0$ $\det(L) = \underbrace{\det(L_1) \cdots \det(L_k)}_{\neq 0}$

$V = \text{span}(RREF(A))$

Each row of A : $\vec{a}_i \in V$

$\Rightarrow$ (Thm in Prof. Gelsa's slides) $\text{span}\{\vec{a}_1, \ldots \vec{a}_n\}$

$\subseteq V$.

Col space(A). $A = \begin{bmatrix} \underbrace{\begin{smallmatrix}2\\1\\0\end{smallmatrix}}_{v_1} & \underbrace{\begin{smallmatrix}3\\2\\0\end{smallmatrix}}_{v_2} & \underbrace{\begin{smallmatrix}1\\1\\0\end{smallmatrix}}_{v_3} \end{bmatrix}$

Basis for the colspace of A.

corresponding columns in A

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{array}{ccc} \text{col 1} & \text{col 2} & \\ \end{array} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

have leading ones

colspace (A) = span $\left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \right\}$

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \underbrace{c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix}}_{\text{single column vector}}$$

$Ex = 0 \quad S = \{ x \mid Ex = 0 \}$ is a solution space to the homogeneous system.

consists of tuples (c_1, c_2, c_3, c_4)

st ☆① $\left[\underbrace{c_1 e_1 + c_2 e_2 + c_3 e_3 + c_4 e_4} = \vec{0} \right.$

$\Leftrightarrow \quad c_1 a_1 + c_2 a_2 + c_3 a_3 + c_4 a_4 = \vec{0}$

(for the same constant $c_1\ c_2\ c_3\ c_4$

similar condition for linear dep. of e_1, e_2, e_3, and e_4.

$$\begin{bmatrix} 1 & 5 & 7 \\ -1 & -4 & -5 \\ 2 & 1 & -4 \end{bmatrix}$$

$$\det \begin{bmatrix} e^t & t & 2+3t \\ e^t & 1 & 3 \\ e^t & 0 & 0 \end{bmatrix} = e^t(\cancel{3t} - 2 - \cancel{3t})$$

$$= 2e^t \neq 0$$

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$$\begin{bmatrix} 3t & |t| \\ 3 & \pm 1 \end{bmatrix} = \begin{cases} 3t - 3t & t > 0 \\ -3t + 3t & t > 0 \end{cases}$$

$$= 0$$

$$\begin{bmatrix} 3t & t \\ 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3t & -t \\ 3 & -1 \end{bmatrix}$$

Suppose $3t = k\,|t| \;\forall t$

$3 = k \qquad t = 1$

$-3 = k \qquad t = -1$