Solution, homogeneous, nonhomogeneous.

coefficients, constants

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 8$$ $(x_1, x_2, \dots x_4)$ vector of unknowns

$$2x_1 + 5x_2 + 10x_3 + 5x_4 = 8$$

Matrix of coefficients. Vector of "system constants"

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 10 & 5 \end{bmatrix} \qquad \begin{bmatrix} 8 \\ 8 \end{bmatrix}$$

Solution $(24+5a-10b, -8-4a+3b, a, b)$ $\forall a, b \in \mathbb{R}$

Means $x_1 = 24 \dots$

Real solution: $(x_1, x_2, x_3, \dots x_n)$

Complex

Many ways to write solution:

$$\begin{array}{c|c} x_1 + x_2 = 3 & x_1 = 1 \\ 3x_1 - 2x_2 = -1 & x_2 = 2 \end{array} \quad \text{or } (1,2)$$

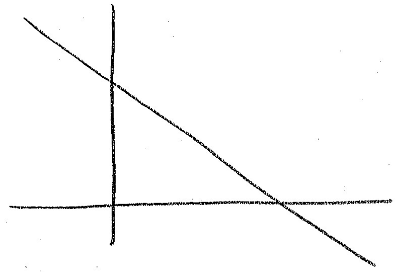
Q: Existence?

How many?

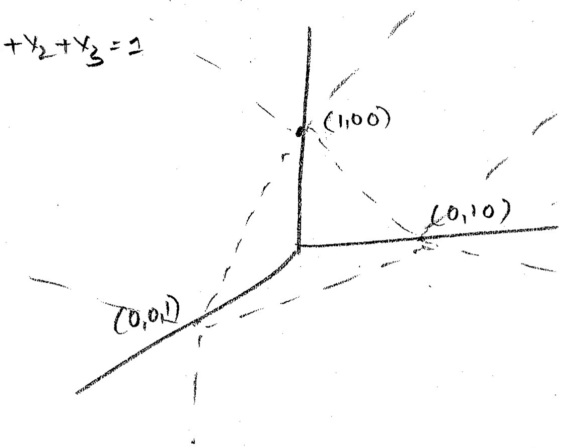
How to determine?

Geometry

$x_1 + x_2 = 3$



$x_1 + x_2 + x_3 = 1$

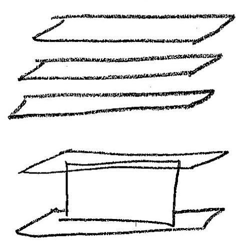


So each equation defines a plane.

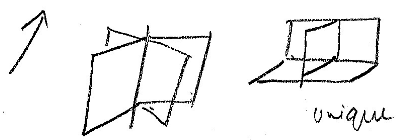
3 equations in 3 unknowns.

↓

defines a plane.



No solutions (No common points in all 3 planes.)



Consistency : at least one solution

Inconsistency : No solution.

Augmented matrix (to be defined next to matrix of coefficients)

Vector formulation

$$-2x_1 + 5x_3 - x_4 = 6$$
$$4x_1 - x_2 + 2x_3 + 2x_4 = -2$$
$$-7x_1 - 6x_2 + 4x_4 = -8$$

$$\begin{bmatrix} -2 & 0 & 5 & -1 \\ 4 & -1 & 2 & 2 \\ -7 & -6 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ -8 \end{bmatrix}$$

We also define the augmented matrix

$$\left[\begin{array}{cccc:c} -2 & 0 & 5 & -1 & -6 \\ 4 & -1 & 2 & 2 & -2 \\ -7 & -6 & 0 & 4 & -8 \end{array}\right]$$

In general

$$A\vec{x} = \vec{b}$$ is a vector eqn.

A is $m \times n$ $\quad \vec{x}$ is an $n \times 1$ column vector

b is an $m \times 1$ column vector.

$\vec{b}$ we may call the "right hand side.

<u>Tuples:</u> set of all real n-tuples $(c_1, \cdots, c_n) = \mathbb{R}^n$

" " complex " " $= \mathbb{C}^n$

Tuples $\equiv$ (Column vectors)^T $\equiv$ Row vectors.

$$(c_1, c_2, \cdots) \equiv \begin{bmatrix} c_1 \\ c_2 \\ \vdots \end{bmatrix}^T \equiv [c_1 \; c_2 \; \cdots]$$

So set of solutions S will be a subset of $\mathbb{R}^n$.

$$A = \begin{bmatrix} 1 & 1 & 2 & -1 \\ 3 & -2 & 1 & 2 \\ 5 & 3 & 3 & -2 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S = \{ x \in \mathbb{R}^4 \mid x = (-t, 4t, t, 5t) \}$$

$-t + 4t + 2t - 5t = 0$ so it satisfies 1st equation.

$$\frac{dx_1}{dt} = \cancel{AA} \; 5t^2 x_1 - 3e^t x_2 + 2\sin t$$

$$\frac{dx_2}{dt} = 6x_1 - 4x_2 + 3t^4$$

$$= \begin{bmatrix} 5t^2 & -3e^t \\ 6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2\sin t \\ 3t^4 \end{bmatrix}$$

$$\frac{d\vec{x}(t)}{dt} = A(t)\, x(t) + b(t)$$

Try Ex 2.3.8 on your own.