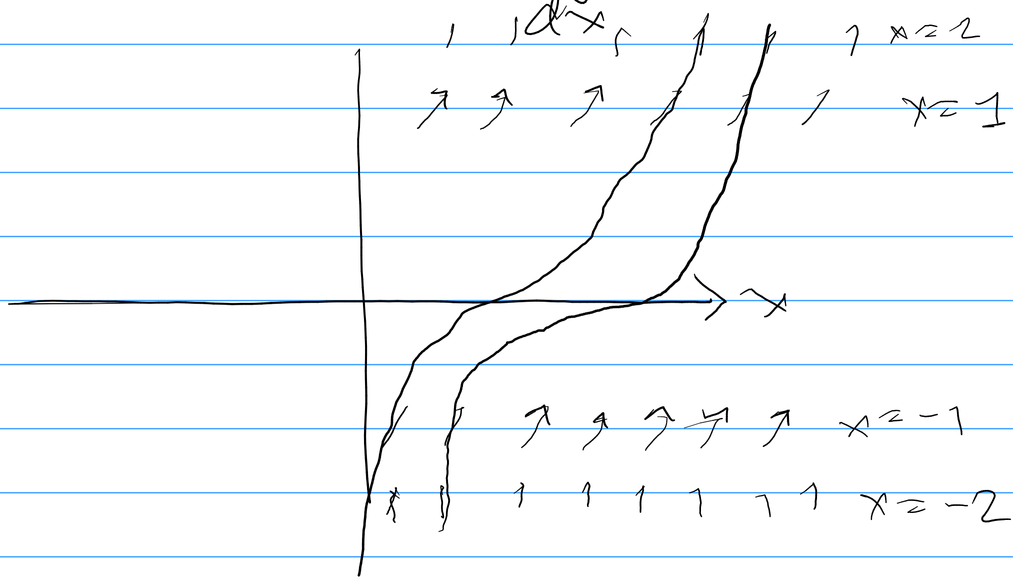
Section 1.3

Slope Fields:

Consider $\frac{dy}{dx} = 2x^2$

Want to interpret $\frac{dy}{dx}$ as the SLOPE of $y(x)$



THERE ARE MANY (∞) solution curves.

They can be obtained by following arrows.

Integrate to get $y(x) - y(0) = \frac{2}{3}x^3$

Or generally $y(x) = \frac{2}{3}x^3 + c$ (where c is some arbitrary constant.

$$\frac{dy}{dx} = y - x \qquad -(1)$$

<u>Equilibrium solutions:</u> There are of the form $y(x) = (\text{constant})\ k$.

Can (1) have an equilibrium solution? NO

$\frac{dy}{dx} = 0 \Rightarrow$ we need $0 = k - x \quad \forall x$

This of course is not possible.

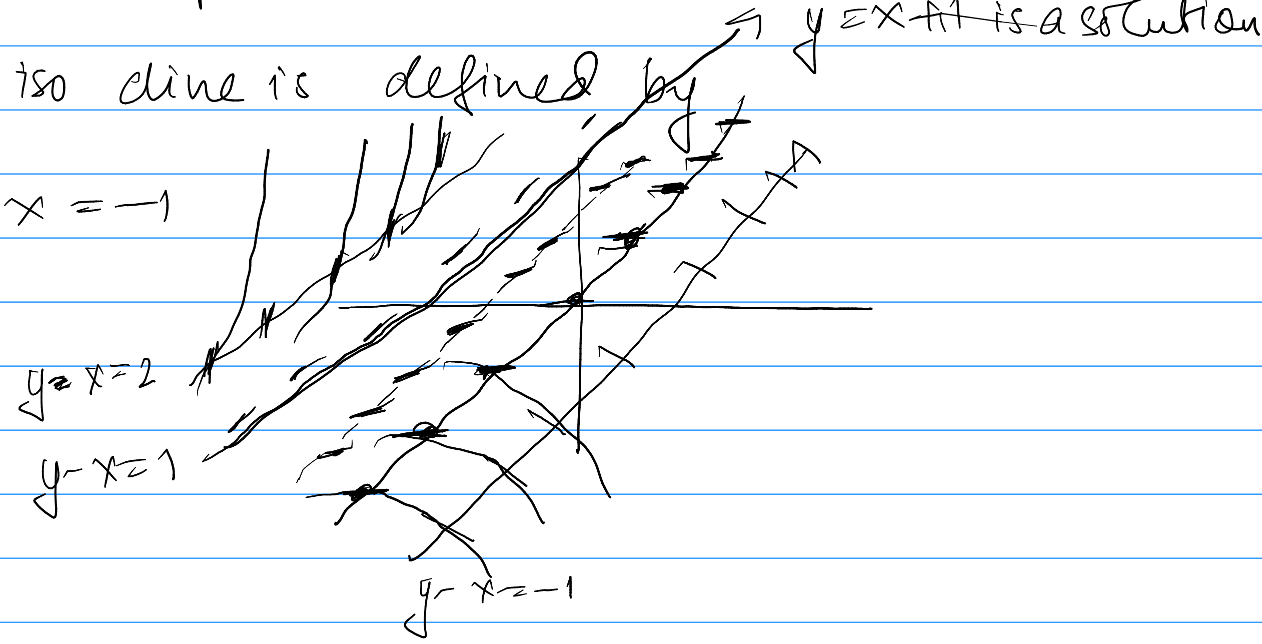
<u>Isoclines:</u> Isoclines are "Lines of constant slope". Iso = constant or equal

Cline = Incline or slope

Lets say the slope is -1

Then the iso cline is defined by

$y - x = -1$



$y - x = 0$

$y - x = 1$

Concavity / Convexity Note that the solution curves that start below the line $y = x+1$ are curved

($y < x+1$)

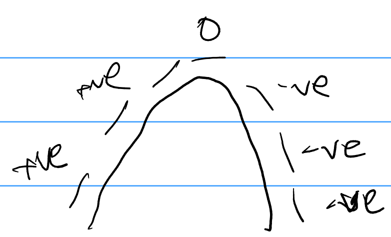
Concave: looks like a cave $\cap$

What tells you about convex / concavity?

2nd derivative. $\frac{d}{dx}\left[\frac{dy}{dx} = y - x\right]$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} - 1 = y - x - 1 < 0 \text{ when } y < x+1$$

when $\frac{d^2y}{dx^2}$ is $-$ve you know $\frac{dy}{dx}$ must be decreasing



Clearly the slope is decreasing and hence the curves must be concave.

Three new concepts

Iso clines
Equilibrium Solutions
Concavity / Convexity

Ended here 1/22/20

Example: Sketch the slope field of

$$\frac{dy}{dx} = y(2-y)$$

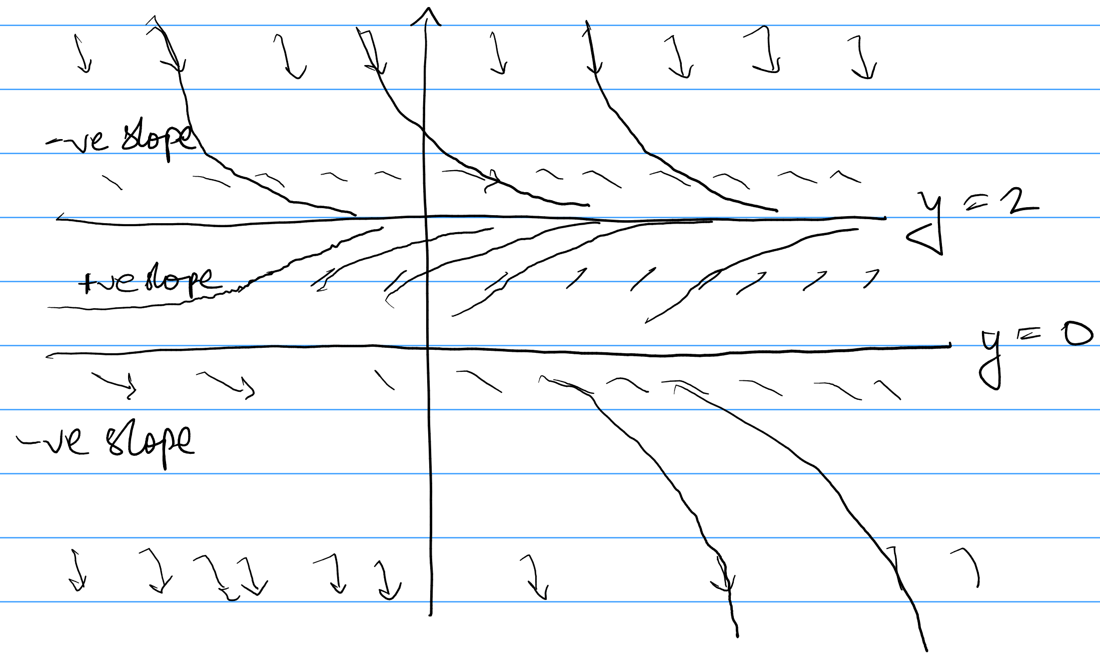
Note that

<0		>0		<0	RHS
	0		2		

Are there any equilibrium solutions:

$$y = k \Rightarrow 0 = k(2-k)$$

Sure! $k=0$ or $k=2$



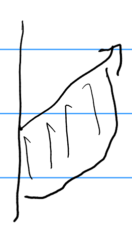
Isoclines: $y(2-y) = k \Rightarrow y^2 - 2y + k = 0$

$$\Rightarrow (y-1)^2 - 1 + k = 0 \qquad y = 1 \pm \sqrt{1-k}$$

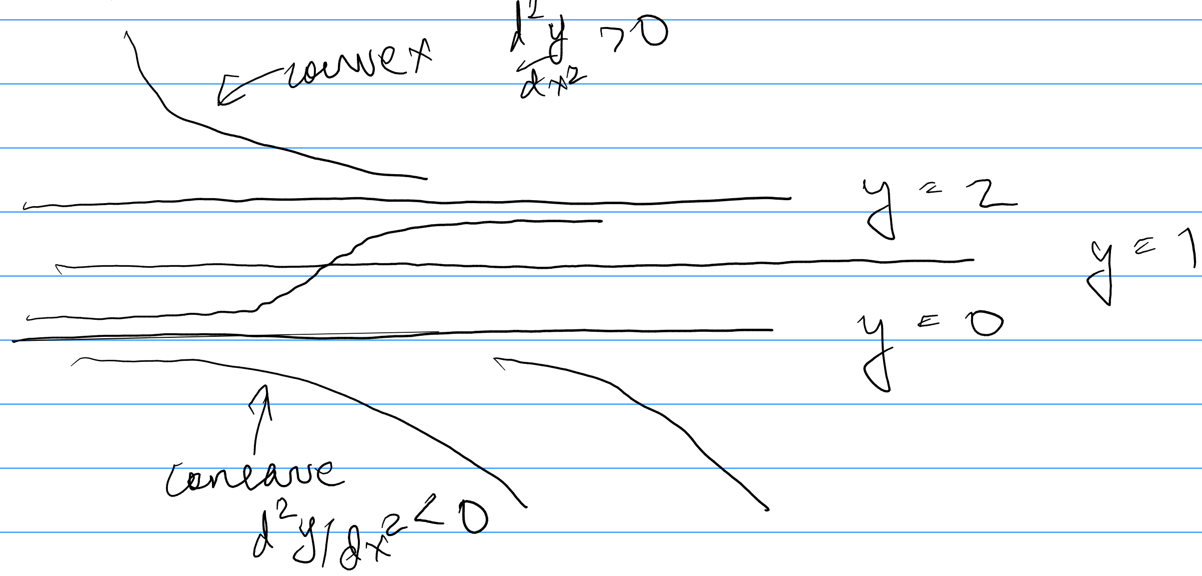
The square root only gives meaningful answers when

$k \leq 1$

So for slopes smaller than 1:



This tallies with our picture above.



Convexity:

$$\frac{dy}{dx} = y(2-y) \Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx}(2-y) - y\frac{dy}{dx}$$

$$= \frac{dy}{dx}2(1-y) = 2y(1-y)(2-y)$$

+ve
2
-ve
1
+ve
0
-ve

signs of $\frac{d^2y}{dx^2}$