lec26

Final exam topics:

1) Seperable equations

$$\frac{dy}{dx} = \frac{7y^2-6y+3}{3x+4} \qquad \int \frac{dy}{7y^2-6y+3} = \int \frac{dx}{3x+4}$$

2) First order linear equations and Integrating factors.

$$\underbrace{\frac{dy}{dx}}_{\text{1st order}} + (3x^2+2)\underbrace{y}_{\text{linear } y} = \sin x$$

↳ Use an integrating factor.

Solving homogeneous and inhomogeneous systems (reduced row echelon form, rank, etc)

$Ax=b$ $\qquad \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{cc|c} 2 & 3 & 0 \\ 1 & 2 & 0 \end{array}\right]$ we pick free variables and write down the general solution.

rank(A) + null(A) = "# of colums of A"

find a basis for colspace(A) ↳ RREF(A)

find a basis ker(A)

Properties of determinants.

$\det(A) = \det(A^T)$

$\det(ABC) = \det(A)\det(B)\det(C)$

$\det(A) = \det(A')$ if $A \sim A'$

$C^3 = AB$ $\quad$ C was invertible $\Leftrightarrow \det(C) \neq 0$

$\det(C^3) = \det(A)\det(B)$

$\det(C)^3 \neq 0 \Rightarrow \det(A)$ and $\det(B) \neq 0$

$\Rightarrow A$ & B invertible.

Ex: If $\det(A) = 3$ find $\det(A^{-1}) = \frac{1}{3}$

$AA^{-1} = I \qquad \det(AA^{-1}) = \det(I) = 1$

$\det(A)\det(A^{-1}) \Rightarrow \det(A^{-1}) = \frac{1}{3}$

Subspaces

1) Determine whether subspace or not

If yes show the 2 closure properties

If no, give an example st one of the closure properties fails.

Ex: $W = \{ p \in P_2 \mid \underbrace{p(1) = p(2)\,p(3)}_{\text{not linear.}} \}$

$p(x) = C$ (constant polynomial)

$p(1) = p(2) = p(3) = C$

$C = C \cdot C \Rightarrow C = 1$ or 0.

$q(x) = 7p(x) = 7$ $\quad q(1) = 7 \neq q(2)q(3) = 49$

$p \in W,\ 7p \notin W \Rightarrow W$ is not a subspace.

Column space | Row space and RREF.

Determine spanning sets.

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Row multiplication

$$\underbrace{[v_1 \cdots v_n]}_{\text{row vector}}{}_{1\times n} \underbrace{\begin{bmatrix} \vec{a}_1 \\ \vdots \\ \vec{a}_n \end{bmatrix}}_{A}{}_{n\times n} = [\text{linear comb of rows}] = [v_1\vec{a}_1 + v_2\vec{a}_2 \cdots + v_n\vec{a}_n]_{1\times n}$$

Rank nullity theorem.

Find basis for range and null space.

Eigenvalues and Eigenvectors.

$A \qquad P(\lambda) = \det(A - \lambda I).$

$= (\lambda - \lambda_1)^{m_1} \cdots (\lambda - \lambda_k)^{m_k}$

Characteristic polynomials and multiplicity of roots.

Eigenspaces. Defective and non defective matrices

General linear transformations

L bases for kernel and range

L generalized rank nullity

n^{th} order constant coeff DEs

L auxiliary polynomial

L general solution

L initial value problems

L inhomogeneous equations

method of annihilators.

Systems of DEs.

Characteristic polynomial

Eigenspaces $E_{\lambda_i} = \ker\{A - \lambda_i I\}$

$\dim(E_{\lambda_i}) = m_i$ for non defective matrices.

$T: P^4 \longrightarrow M_{2\times 2}$

$T(ax^4 + bx^3 + cx^2 + dx + e)$

$$= \begin{bmatrix} a+b-c & c \\ d+e & 2e \end{bmatrix}$$

$\dim(\ker(T)) + \dim(\operatorname{ran}(T)) = \underbrace{\text{dim of the domain}}^{5}$ space of T.

$$y''' + 3y'' - 7y' + 4y = \sin x + e^{7x}$$

$P(x) = x^3 + 3x^2 - 7x + 4$ = factorize this and find 3 linearly indep. solutions.

y_h, $y_p \leftarrow$ a particular solution.

$A(D)(\sin x + e^{7x})$.

real eigenvalues complex evals.