Theu

By S.L.L.N. For stationary sequences, if it is
stationary and
$$\mathbb{E}(\frac{1}{2t}) < t_{\infty}$$
, then
 $\lim_{t \to t_{\infty}} \frac{1}{t} (\frac{1}{2t} + -t_{\infty}) = \lim_{t \to t_{\infty}} \frac{x_{ot}}{t}$
exist with prob L.
Stationarity of it's (\mathbb{T}) (when (x_{st}) addition)
Lef x_{st} be a s.p. satisfying (\mathbb{D}, \mathbb{T}) , and (\mathbb{S})
 $\Rightarrow \mathbb{E}(x_{st}) = (\frac{1}{2s-t})$

From
$$1$$
:
 $\left| \frac{f_{x+p}}{f_{x+p}} \leq \frac{f_{x}}{f_{x}} + \frac{f_{p}}{f_{p}} \right|$

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 $\left| \frac{f_{x}}{f_{x}} \right|^{2} = \frac{f_{x}}{f_{x}} + \frac{f_{p}}{f_{p}} = \frac{f_{p}}{f_{p}}$

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where
$$r = i \cdot r + \frac{g_{\alpha}}{\alpha}$$

Theoreme (Kingman's Subadditive Esgodic Theorem)

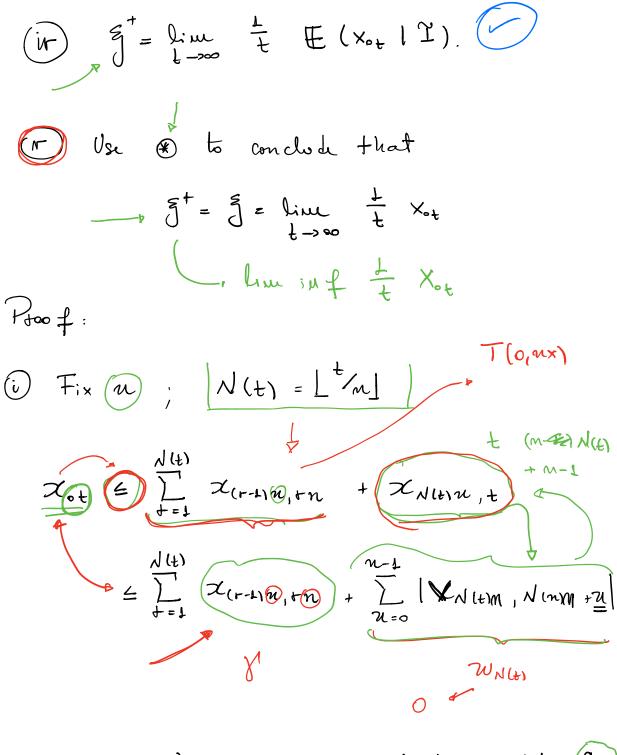
Let x_{st} be a stationary subadditure process with time constant of them line $\frac{x_{ot}}{t}$ of t $t \rightarrow \infty$ t texist almost surger, If we write $\frac{z}{2} = \lim \frac{x_{ot}}{t}$ then E(z) = 0.

Also

$$= \mathbb{E} \left| \frac{X_{\text{ot}}}{t} - \frac{g}{J} \right| \xrightarrow{t \to \infty} 0 \quad (3)$$

thus $x_{ot}/t \longrightarrow g$ in prob. Moreover if Y is the τ -alg. of events defined in terms of (π_{st}) and invariants under the map $\chi_{st} \xrightarrow{T} \chi_{st}$, term

+ her $= \lim_{L \to \infty} \frac{L}{t} = \left(x_{ot} \mid 1^{2} \right).$ In pasticular if <u>Y</u> is trivial (I ergodic) the z = y. I the group of D depend on a decomposition $\xrightarrow{}$ ($x_{st} = J_{st} + J_{st}$) with Let additive process, E(Jor) = I and 3st nonnegative subadditive process with time constant = 0 . $\int \mathcal{L} = \left(\Theta(s,t) \in \Theta(s,u) + \Theta(u,t) \right)$ Ofganization of the proof: (i) $z^{+} = \lim_{t \to \infty} \sup_{t \to \infty} \frac{x_{0t}}{t}$ is a.s. finite. $E(s^+) = s^-$ (ii) $(ii) \quad \mathbb{E}\left(\frac{x_{i}}{L} - \tilde{z}^{*}\right) \quad --- \tilde{z}$ 0



· (X(+-s)n, +n) stationary with finite expectation Ju By S.L.L.N the following limit exist a.s.

$$\lim_{N \to \infty} \frac{1}{N} \prod_{t=1}^{N} \chi_{t-tn} = 3n.$$

and
$$(\underline{E}(\underline{J}_{M}) = \underline{J}_{M})$$

For each $\underline{z} > 0$
 $\sum_{N=1}^{\infty} P(\underline{w}_{N} / \underline{N} \geq \underline{z}) = \sum_{N=1}^{\infty} P(\underline{w}_{0} \geq \underline{N} \underline{z})$
 $\leq \underline{z} E(\underline{w}_{0}) \leq 0$

By Bosel - Cantelli

$$- - D live \frac{NN/N}{N} = 0$$
 a.s.
 $\frac{N-200}{N}$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{m_{i}} = \frac{3m_{i}}{m_{i}} + \sum_{j=1}^{n} \frac{1}{m_{i}} = \frac{3m_{i}}{m_{i}} + \sum_{j=1}^{n} \frac{1}{m_{i}} + \sum_{j=1}^{n} \frac{1}$$

=>
$$j^{+} <+\infty$$
 a.s. and $\mathbb{E}(j) \in \mathbb{O}_{m} = \mathbb{E}(S_{n})$
for every n
=> $\mathbb{E}(j^{+}) = \mathbb{N}$
To prove $\mathbb{E}(j^{+}) \ge \mathbb{N}$
 $\cdot [a_{st} = \mathbb{L}_{t=st}^{t} \times_{t-t}, t]$
 $a_{st} \to additive and $\mathbb{X}_{st} \in a_{st}$
 $h_{st} \to additive and $\mathbb{X}_{st} \in a_{st}$
 $h_{st} = a_{st} - \mathbb{X}_{st}$
 $h_{st} = b_{st} f = b_{st}/f \rightarrow \mathbb{I}_{st}$
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 $h_{st} = b_{st} f = b_{st}/f = b_{st} = b_{st} + b_{st}/f = b_{st} = b_{st} - b_{st}/f = b_{st} = b_{st} - b_{st}/f = b_{st} = b_{st} - b_{st}/f = b_{st} = b_{st} + b_{st}/f = b_{st} = b_{st} - b_{st}/f = b_{st} = b_{st} = b_{st} = b_{st} - b_{st}/f = b_{st} = b_{st} - b_{st}/f = b_{st} = b_{st} = b_{st} - b_{st}$$$

$$E_{y} \text{ necestory convergence}$$

$$\frac{\lim_{M \to \infty} E(B_{M}) = E(\lim_{M \to \infty} B_{M})$$

$$= \underbrace{E(S_{k} - \underline{g}^{*})}_{g_{k} - E(\underline{g}^{*})}$$

$$= \underbrace{g_{k} - E(\underline{g}^{*})}_{M \to \infty} E(\underbrace{b_{0M}}{n})$$

$$= \underbrace{\lim_{M \to \infty} E(B_{0})(\underline{e}) \underbrace{\lim_{M \to \infty} E(b_{0M}/n)}_{M \to \infty} = \underbrace{g_{k} - g_{k}}_{g_{k}}$$

$$= \underbrace{g_{k} - M}_{g_{k}}$$

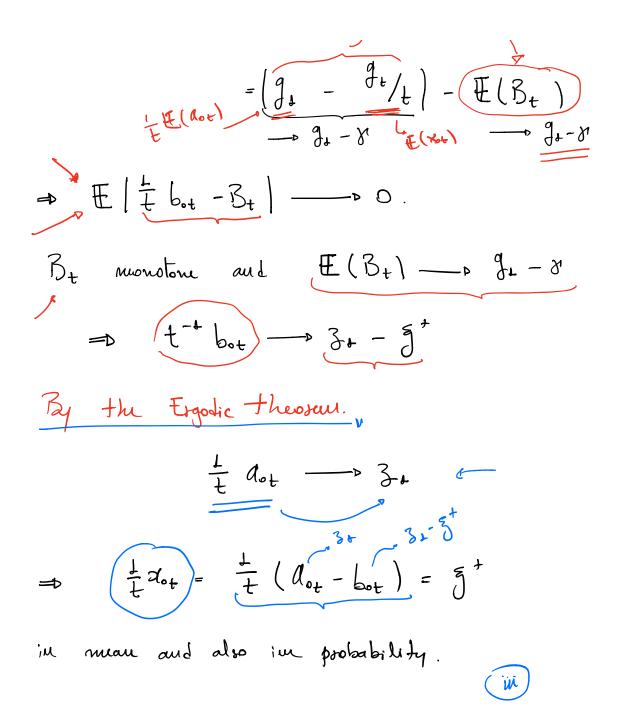
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(F)
$$\times_{st} = \int_{st} + \int_{st} - p$$

 $\int_{sta. add.} I(f_{ot}) = f.$
(S) $= \lim_{t \to \infty} \sup_{s \to t/t} i$ $I(f_{ot}) = f.$
 $\int_{t \to \infty} I(t = 0.)$ (i)
 $\int_{t \to \infty} \int_{st} f(t = 0.)$ (j) $\int_{st} f(t = 0.)$ (j)