

FORM GR9768

68

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MATHEMATICS TEST (RESCALED)



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Princeton, N.J. 08541

MATHEMATICS TEST (RESCALED)

Time—170 minutes

66 Questions

**Directions:** Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is the best of the choices offered and then mark the corresponding space on the answer sheet.

Computation and scratchwork may be done in this examination book.

**Note:** In this examination:

- (1) All logarithms with an unspecified base are natural logarithms (that is, with base  $e$ ).
  - (2) The set of all real numbers  $x$  such that  $a \leq x \leq b$  is denoted by  $[a, b]$ .
  - (3) The symbols  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$ , and  $\mathbb{C}$  denote the sets of integers, rational numbers, real numbers, and complex numbers, respectively.
- 

1. If  $F(x) = \int_e^x \log t \, dt$  for all positive  $x$ , then  $F'(x) =$

- (A)  $x$
  - (B)  $\frac{1}{x}$
  - (C)  $\log x$
  - (D)  $x \log x$
  - (E)  $x \log x - 1$
- 

2. If  $F(1) = 2$  and  $F(n) = F(n - 1) + \frac{1}{2}$  for all integers  $n > 1$ , then  $F(101) =$

- (A) 49
  - (B) 50
  - (C) 51
  - (D) 52
  - (E) 53
- 

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3. If  $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$  is invertible under matrix multiplication, then its inverse is

(A)  $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

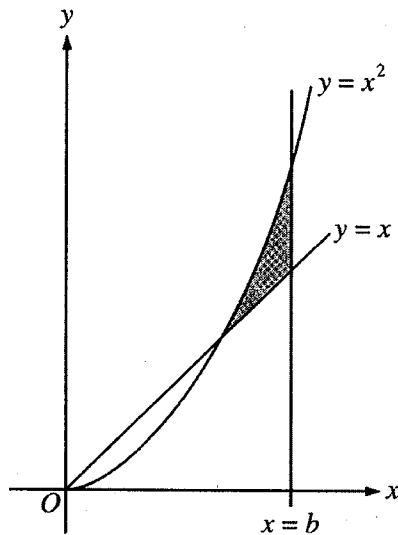
(B)  $\frac{1}{a^2 + b^2} \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(C)  $\frac{1}{a^2 + b^2} \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(D)  $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(E)  $\frac{1}{a^2 - b^2} \begin{pmatrix} -b & a \\ a & b \end{pmatrix}$

---



4. If  $b > 0$  and if  $\int_0^b x \, dx = \int_0^b x^2 \, dx$ , then the area of the shaded region in the figure above is

(A)  $\frac{1}{12}$

(B)  $\frac{1}{6}$

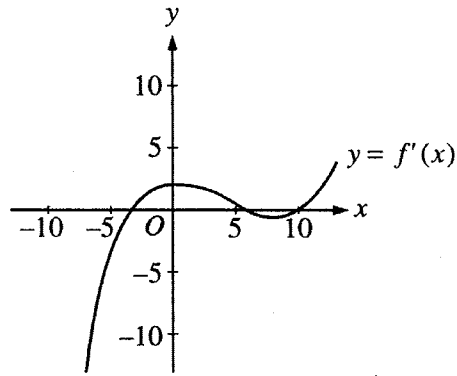
(C)  $\frac{1}{4}$

(D)  $\frac{1}{3}$

(E)  $\frac{1}{2}$

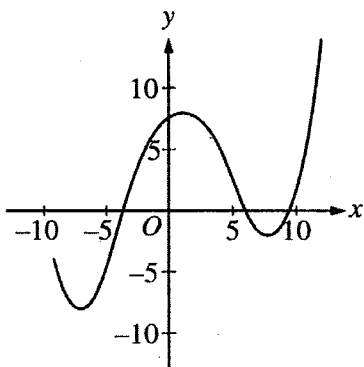
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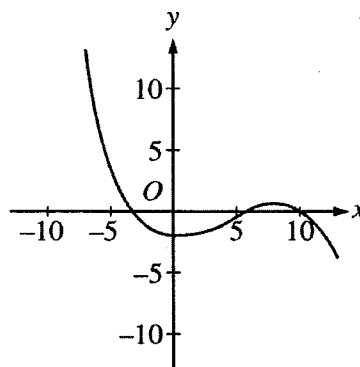


5. If the figure above is the graph of  $y = f'(x)$ , which of the following could be the graph of  $y = f(x)$  ?

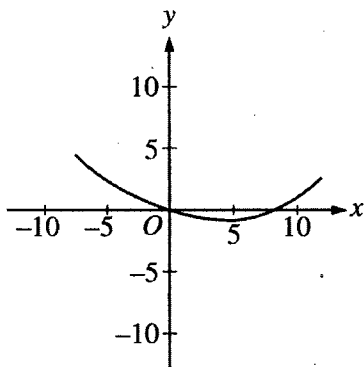
(A)



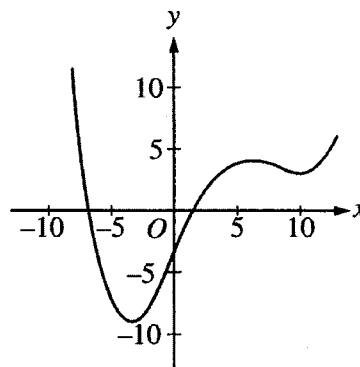
(D)



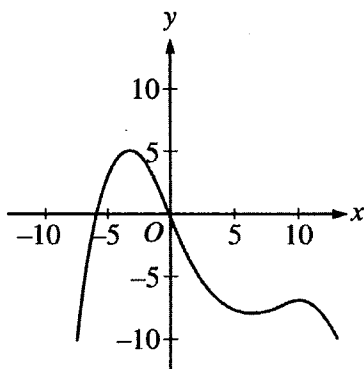
(B)



(E)



(C)



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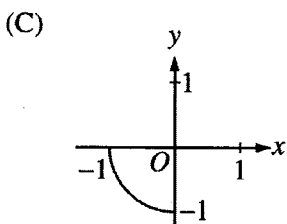
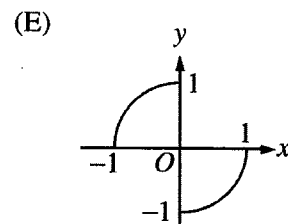
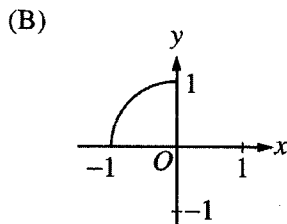
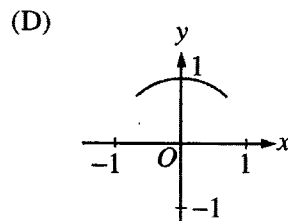
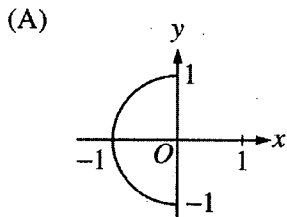
6. Consider the following sequence of instructions.

1. Set  $k = 999$ ,  $i = 1$ , and  $p = 0$ .
2. If  $k > i$ , then go to step 3; otherwise go to step 5.
3. Replace  $i$  with  $2i$  and replace  $p$  with  $p + 1$ .
4. Go to step 2.
5. Print  $p$ .

If these instructions are followed, what number will be printed at step 5?

- (A) 1
- (B) 2
- (C) 10
- (D) 512
- (E) 999

7. Which of the following indicates the graph of  $\{(\sin t, \cos t) : -\frac{\pi}{2} \leq t \leq 0\}$  in the  $xy$ -plane?



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8.  $\int_0^1 \frac{x}{1+x^2} dx =$

- (A) 1
  - (B)  $\frac{\pi}{4}$
  - (C)  $\tan^{-1} \frac{\sqrt{2}}{2}$
  - (D)  $\log 2$
  - (E)  $\log \sqrt{2}$
- 

9. If  $S$  is a nonempty finite set with  $k$  elements, then the number of one-to-one functions from  $S$  onto  $S$  is

- (A)  $k!$
  - (B)  $k^2$
  - (C)  $k^k$
  - (D)  $2^k$
  - (E)  $2^{k+1}$
- 

10. Let  $g$  be the function defined on the set of all real numbers by

$$g(x) = \begin{cases} 1 & \text{if } x \text{ is rational,} \\ e^x & \text{if } x \text{ is irrational.} \end{cases}$$

Then the set of numbers at which  $g$  is continuous is

- (A) the empty set
  - (B)  $\{0\}$
  - (C)  $\{1\}$
  - (D) the set of rational numbers
  - (E) the set of irrational numbers
- 

11. For all real numbers  $x$  and  $y$ , the expression  $\frac{x+y+|x-y|}{2}$  is equal to

- (A) the maximum of  $x$  and  $y$
  - (B) the minimum of  $x$  and  $y$
  - (C)  $|x+y|$
  - (D) the average of  $|x|$  and  $|y|$
  - (E) the average of  $|x+y|$  and  $x-y$
- 

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12. Let  $B$  be a nonempty bounded set of real numbers and let  $b$  be the least upper bound of  $B$ . If  $b$  is not a member of  $B$ , which of the following is necessarily true?

- (A)  $B$  is closed.
  - (B)  $B$  is not open.
  - (C)  $b$  is a limit point of  $B$ .
  - (D) No sequence in  $B$  converges to  $b$ .
  - (E) There is an open interval containing  $b$  but containing no point of  $B$ .
- 

13. A drawer contains 2 blue, 4 red, and 2 yellow socks. If 2 socks are to be randomly selected from the drawer, what is the probability that they will be the same color?

- (A)  $\frac{2}{7}$
  - (B)  $\frac{2}{5}$
  - (C)  $\frac{3}{7}$
  - (D)  $\frac{1}{2}$
  - (E)  $\frac{3}{5}$
- 

14. Let  $\mathbb{R}$  be the set of real numbers and let  $f$  and  $g$  be functions from  $\mathbb{R}$  into  $\mathbb{R}$ . The negation of the statement

“For each  $s$  in  $\mathbb{R}$ , there exists an  $r$  in  $\mathbb{R}$  such that if  $f(r) > 0$ , then  $g(s) > 0$ .”

is which of the following?

- (A) For each  $s$  in  $\mathbb{R}$ , there does not exist an  $r$  in  $\mathbb{R}$  such that if  $f(r) > 0$ , then  $g(s) > 0$ .
  - (B) For each  $s$  in  $\mathbb{R}$ , there exists an  $r$  in  $\mathbb{R}$  such that  $f(r) > 0$  and  $g(s) \leq 0$ .
  - (C) There exists an  $s$  in  $\mathbb{R}$  such that for each  $r$  in  $\mathbb{R}$ ,  $f(r) > 0$  and  $g(s) \leq 0$ .
  - (D) There exists an  $s$  in  $\mathbb{R}$  and there exists an  $r$  in  $\mathbb{R}$  such that  $f(r) \leq 0$  and  $g(s) \leq 0$ .
  - (E) For each  $r$  in  $\mathbb{R}$ , there exists an  $s$  in  $\mathbb{R}$  such that  $f(r) \leq 0$  and  $g(s) \leq 0$ .
- 

15. If  $g$  is a function defined on the open interval  $(a, b)$  such that  $a < g(x) < x$  for all  $x \in (a, b)$ , then  $g$  is

- (A) an unbounded function
  - (B) a nonconstant function
  - (C) a nonnegative function
  - (D) a strictly increasing function
  - (E) a polynomial function of degree 1
- 

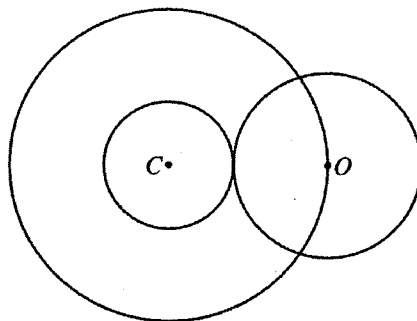
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16. For what value (or values) of  $m$  is the vector  $(1, 2, m, 5)$  a linear combination of the vectors  $(0, 1, 1, 1)$ ,  $(0, 0, 0, 1)$ , and  $(1, 1, 2, 0)$ ?
- (A) For no value of  $m$   
 (B)  $-1$  only  
 (C)  $1$  only  
 (D)  $3$  only  
 (E) For infinitely many values of  $m$
- 

17. For a function  $f$ , the finite differences  $\Delta f(x)$  and  $\Delta^2 f(x)$  are defined by  $\Delta f(x) = f(x + 1) - f(x)$  and  $\Delta^2 f(x) = \Delta f(x + 1) - \Delta f(x)$ . What is the value of  $f(4)$ , given the following partially completed finite difference table?

$x$	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
1	$-1$	$4$	
2		$-2$	$6$
3			
4			

- (A)  $-5$   
 (B)  $-1$   
 (C)  $1$   
 (D)  $3$   
 (E)  $5$
- 



18. In the figure above, the annulus with center  $C$  has inner radius  $r$  and outer radius  $1$ . As  $r$  increases, the circle with center  $O$  contracts and remains tangent to the inner circle. If  $A(r)$  is the area of the annulus and  $a(r)$  is the area of the circular region with center  $O$ , then  $\lim_{r \rightarrow 1^-} \frac{A(r)}{a(r)} =$

- (A)  $0$   
 (B)  $\frac{2}{\pi}$   
 (C)  $1$   
 (D)  $\frac{\pi}{2}$   
 (E)  $\infty$
- 

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19. Which of the following are multiplication tables for groups with four elements?

I. 

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>b</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>a</i>
<i>c</i>	<i>c</i>	<i>d</i>	<i>a</i>	<i>b</i>
<i>d</i>	<i>d</i>	<i>a</i>	<i>b</i>	<i>c</i>

II. 

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>b</i>	<i>b</i>	<i>a</i>	<i>d</i>	<i>c</i>
<i>c</i>	<i>c</i>	<i>d</i>	<i>a</i>	<i>a</i>
<i>d</i>	<i>d</i>	<i>c</i>	<i>a</i>	<i>b</i>

III. 

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>b</i>	<i>b</i>	<i>a</i>	<i>d</i>	<i>c</i>
<i>c</i>	<i>c</i>	<i>d</i>	<i>c</i>	<i>d</i>
<i>d</i>	<i>d</i>	<i>c</i>	<i>d</i>	<i>c</i>

- (A) None
- (B) I only
- (C) I and II only
- (D) II and III only
- (E) I, II, and III

20. Which of the following statements are true for every function  $f$ , defined on the set of all real numbers,

such that  $\lim_{x \rightarrow 0} \frac{f(x)}{x}$  is a real number  $L$  and  $f(0) = 0$ ?

- I.  $f$  is differentiable at 0.
- II.  $L = 0$
- III.  $\lim_{x \rightarrow 0} f(x) = 0$

- (A) None
- (B) I only
- (C) III only
- (D) I and III only
- (E) I, II, and III

21. What is the area of the region bounded by the coordinate axes and the line tangent to the graph of

$y = \frac{1}{8}x^2 + \frac{1}{2}x + 1$  at the point  $(0, 1)$ ?

- (A)  $\frac{1}{16}$
- (B)  $\frac{1}{8}$
- (C)  $\frac{1}{4}$
- (D) 1
- (E) 2

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22. Let  $\mathbb{Z}$  be the group of all integers under the operation of addition. Which of the following subsets of  $\mathbb{Z}$  is NOT a subgroup of  $\mathbb{Z}$  ?

- (A)  $\{0\}$
  - (B)  $\{n \in \mathbb{Z} : n \geq 0\}$
  - (C)  $\{n \in \mathbb{Z} : n \text{ is an even integer}\}$
  - (D)  $\{n \in \mathbb{Z} : n \text{ is divisible by both 6 and 9}\}$
  - (E)  $\mathbb{Z}$
- 

23. In the Euclidean plane, point  $A$  is on a circle centered at point  $O$ , and  $O$  is on a circle centered at  $A$ . The circles intersect at points  $B$  and  $C$ . What is the measure of angle  $BAC$  ?

- (A)  $60^\circ$
  - (B)  $90^\circ$
  - (C)  $120^\circ$
  - (D)  $135^\circ$
  - (E)  $150^\circ$
- 

24. Which of the following sets of vectors is a basis for the subspace of Euclidean 4-space consisting of all vectors that are orthogonal to both  $(0, 1, 1, 1)$  and  $(1, 1, 1, 0)$  ?

- (A)  $\{(0, -1, 1, 0)\}$
  - (B)  $\{(1, 0, 0, 0), (0, 0, 0, 1)\}$
  - (C)  $\{(-2, 1, 1, -2), (0, 1, -1, 0)\}$
  - (D)  $\{(1, -1, 0, 1), (-1, 1, 0, -1), (0, 1, -1, 0)\}$
  - (E)  $\{(0, 0, 0, 0), (-1, 1, 0, -1), (0, 1, -1, 0)\}$
- 

25. Let  $f$  be the function defined by  $f(x, y) = 5x - 4y$  on the region in the  $xy$ -plane satisfying the inequalities  $x \leq 2$ ,  $y \geq 0$ ,  $x + y \geq 1$ , and  $y - x \leq 0$ . The maximum value of  $f$  on this region is

- (A) 1
  - (B) 2
  - (C) 5
  - (D) 10
  - (E) 15
- 

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26. Let  $f$  be the function defined by

$$f(x) = \begin{cases} -x^2 + 4x - 2 & \text{if } x < 1, \\ -x^2 + 2 & \text{if } x \geq 1. \end{cases}$$

Which of the following statements about  $f$  is true?

- (A)  $f$  has an absolute maximum at  $x = 0$ .
  - (B)  $f$  has an absolute maximum at  $x = 1$ .
  - (C)  $f$  has an absolute maximum at  $x = 2$ .
  - (D)  $f$  has no absolute maximum.
  - (E)  $f$  has local maxima at both  $x = 0$  and  $x = 2$ .
- 

27. Let  $f$  be a function such that  $f(x) = f(1 - x)$  for all real numbers  $x$ . If  $f$  is differentiable everywhere, then  $f'(0) =$

- (A)  $f(0)$
  - (B)  $f(1)$
  - (C)  $-f(0)$
  - (D)  $f'(1)$
  - (E)  $-f'(1)$
- 

28. If  $V_1$  and  $V_2$  are 6-dimensional subspaces of a 10-dimensional vector space  $V$ , what is the smallest possible dimension that  $V_1 \cap V_2$  can have?

- (A) 0
  - (B) 1
  - (C) 2
  - (D) 4
  - (E) 6
- 

29. Assume that  $p$  is a polynomial function on the set of real numbers. If  $p(0) = p(2) = 3$  and

$$p'(0) = p'(2) = -1, \text{ then } \int_0^2 xp''(x) dx =$$

- (A) -3
  - (B) -2
  - (C) -1
  - (D) 1
  - (E) 2
- 

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30. Suppose  $B$  is a basis for a real vector space  $V$  of dimension greater than 1. Which of the following statements could be true?

- (A) The zero vector of  $V$  is an element of  $B$ .
  - (B)  $B$  has a proper subset that spans  $V$ .
  - (C)  $B$  is a proper subset of a linearly independent subset of  $V$ .
  - (D) There is a basis for  $V$  that is disjoint from  $B$ .
  - (E) One of the vectors in  $B$  is a linear combination of the other vectors in  $B$ .
- 

31. Which of the following CANNOT be a root of a polynomial in  $x$  of the form  $9x^5 + ax^3 + b$ , where  $a$  and  $b$  are integers?

- (A)  $-9$
  - (B)  $-5$
  - (C)  $\frac{1}{4}$
  - (D)  $\frac{1}{3}$
  - (E)  $9$
- 

32. When 20 children in a classroom line up for lunch, Pat insists on being somewhere ahead of Lynn. If Pat's demand is to be satisfied, in how many ways can the children line up?

- (A)  $20!$
  - (B)  $19!$
  - (C)  $18!$
  - (D)  $\frac{20!}{2}$
  - (E)  $20 \cdot 19$
- 

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33. How many integers from 1 to 1,000 are divisible by 30 but not by 16 ?

- (A) 29
  - (B) 31
  - (C) 32
  - (D) 33
  - (E) 38
- 

34. Suppose  $f$  is a differentiable function for which  $\lim_{x \rightarrow \infty} f(x)$  and  $\lim_{x \rightarrow \infty} f'(x)$  both exist and are finite. Which of the following must be true?

- (A)  $\lim_{x \rightarrow \infty} f'(x) = 0$
  - (B)  $\lim_{x \rightarrow \infty} f''(x) = 0$
  - (C)  $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} f'(x)$
  - (D)  $f$  is a constant function.
  - (E)  $f'$  is a constant function.
- 

35. In  $xyz$ -space, an equation of the plane tangent to the surface  $z = e^{-x} \sin y$  at the point where  $x = 0$  and  $y = \frac{\pi}{2}$  is

- (A)  $x + y = 1$
  - (B)  $x + z = 1$
  - (C)  $x - z = 1$
  - (D)  $y + z = 1$
  - (E)  $y - z = 1$
- 

36. For each real number  $x$ , let  $\mu(x)$  be the mean of the numbers 4, 9, 7, 5, and  $x$ ; and let  $\eta(x)$  be the median of these five numbers. For how many values of  $x$  is  $\mu(x) = \eta(x)$  ?

- (A) None
  - (B) One
  - (C) Two
  - (D) Three
  - (E) Infinitely many
- 

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37.  $\sum_{k=1}^{\infty} \frac{k^2}{k!} =$

- (A)  $e$
  - (B)  $2e$
  - (C)  $(e + 1)(e - 1)$
  - (D)  $e^2$
  - (E)  $\infty$
- 

38. Which of the following integrals on the interval  $\left[0, \frac{\pi}{4}\right]$  has the greatest value?

- (A)  $\int_0^{\frac{\pi}{4}} \sin t \, dt$
  - (B)  $\int_0^{\frac{\pi}{4}} \cos t \, dt$
  - (C)  $\int_0^{\frac{\pi}{4}} \cos^2 t \, dt$
  - (D)  $\int_0^{\frac{\pi}{4}} \cos 2t \, dt$
  - (E)  $\int_0^{\frac{\pi}{4}} \sin t \cos t \, dt$
- 

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39. Consider the function  $f$  defined by  $f(x) = e^{-x}$  on the interval  $[0, 10]$ . Let  $n > 1$  and let  $x_0, x_1, \dots, x_n$  be numbers such that  $0 = x_0 < x_1 < x_2 < \dots < x_{n-1} < x_n = 10$ . Which of the following is greatest?

- (A)  $\sum_{j=1}^n f(x_j)(x_j - x_{j-1})$   
(B)  $\sum_{j=1}^n f(x_{j-1})(x_j - x_{j-1})$   
(C)  $\sum_{j=1}^n f\left(\frac{x_j + x_{j-1}}{2}\right)(x_j - x_{j-1})$   
(D)  $\int_0^{10} f(x) dx$   
(E) 0
- 

40. A fair coin is to be tossed 8 times. What is the probability that more of the tosses will result in heads than will result in tails?

- (A)  $\frac{1}{4}$   
(B)  $\frac{1}{3}$   
(C)  $\frac{87}{256}$   
(D)  $\frac{23}{64}$   
(E)  $\frac{93}{256}$
- 

41. The function  $f(x, y) = xy - x^3 - y^3$  has a relative maximum at the point

- (A)  $(0, 0)$   
(B)  $(1, 1)$   
(C)  $(-1, -1)$   
(D)  $(1, 3)$   
(E)  $\left(\frac{1}{3}, \frac{1}{3}\right)$
- 

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42. Consider the points  $A = (-1, 2)$ ,  $B = (6, 4)$ , and  $C = (1, -20)$  in the plane. For how many different points  $D$  in the plane are  $A$ ,  $B$ ,  $C$ , and  $D$  the vertices of a parallelogram?

- (A) None
  - (B) One
  - (C) Two
  - (D) Three
  - (E) Four
- 

43. If  $A$  is a  $3 \times 3$  matrix such that  $A \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$  and  $A \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ , then the product  $A \begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix}$  is

(A)  $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(B)  $\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$

(C)  $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

(D)  $\begin{pmatrix} 9 \\ 10 \\ 11 \end{pmatrix}$

(E) not uniquely determined by the information given

---

44. Let  $f$  denote the function defined for all  $x > 0$  by  $f(x) = (\sqrt{x})^x$ . Which of the following statements is FALSE?

(A)  $\lim_{x \rightarrow 0^+} f(x) = 1$

(B)  $\lim_{x \rightarrow \infty} f(x) = \infty$

(C)  $f(x) = x^{x/2}$  for all  $x > 0$ .

(D) The derivative  $f'(x)$  is positive for all  $x > 0$ .

(E) The derivative  $f'(x)$  is increasing for all  $x > 0$ .

---

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45. An experimental car is found to have a fuel efficiency  $E(v)$ , in miles per gallon of fuel, where  $v$  is the speed of the car, in miles per hour. For a certain 4-hour trip, if  $v = v(t)$  is the speed of the car  $t$  hours after the trip started, which of the following integrals represents the number of gallons of fuel that the car used on the trip?

(A)  $\int_0^4 \frac{v(t)}{E(v(t))} dt$

(B)  $\int_0^4 \frac{E(v(t))}{v(t)} dt$

(C)  $\int_0^4 \frac{tv(t)}{E(v(t))} dt$

(D)  $\int_0^4 \frac{tE(v(t))}{v(t)} dt$

(E)  $\int_0^4 v(t)E(v(t)) dt$

---

46. For  $0 < t < \pi$ , the matrix  $\begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$  has distinct complex eigenvalues  $\lambda_1$  and  $\lambda_2$ . For what value of  $t$ ,  $0 < t < \pi$ , is  $\lambda_1 + \lambda_2 = 1$ ?

(A)  $\frac{\pi}{6}$

(B)  $\frac{\pi}{4}$

(C)  $\frac{\pi}{3}$

(D)  $\frac{\pi}{2}$

(E)  $\frac{2\pi}{3}$

---

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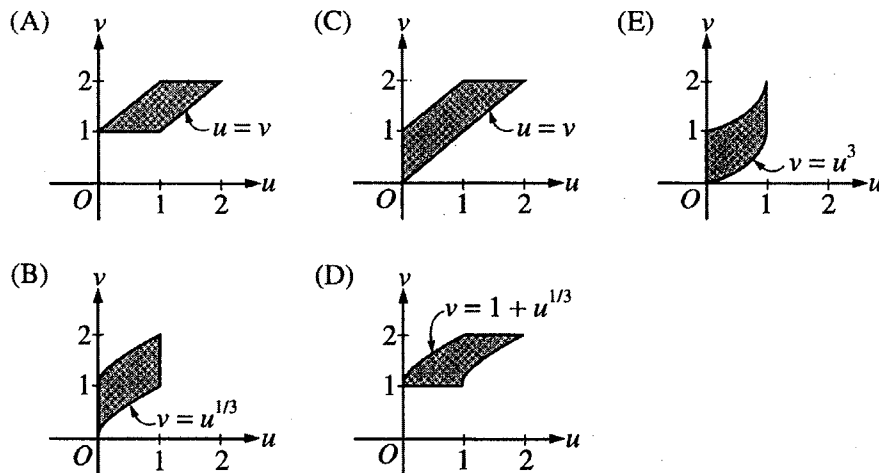
47. Let  $x$  and  $y$  be uniformly distributed, independent random variables on  $[0, 1]$ . The probability that the distance between  $x$  and  $y$  is less than  $\frac{1}{2}$  is

- (A)  $\frac{1}{4}$
- (B)  $\frac{1}{3}$
- (C)  $\frac{1}{2}$
- (D)  $\frac{2}{3}$
- (E)  $\frac{3}{4}$

48. Consider the change of variables from the  $xy$ -plane to the  $uv$ -plane given by the equations

$$\begin{aligned} u &= x^{1/3} + y \\ v &= 1 + y. \end{aligned}$$

Under this transformation, the image of the region  $\{(x, y): 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$  is which of the following shaded regions?



GO ON TO THE NEXT PAGE.

49. If  $f$  is a continuous function on the set of real numbers and if  $a$  and  $b$  are real numbers, which of the following must be true?

I.  $\int_a^b f(x) dx = \int_{a+3}^{b+3} f(x-3) dx$

II.  $\int_a^b f(x) dx = \int_a^3 f(x) dx - \int_b^3 f(x) dx$

III.  $\int_{3a}^{3b} f(x) dx = 3 \int_a^b f(3x) dx$

- (A) I only
  - (B) II only
  - (C) I and II only
  - (D) II and III only
  - (E) I, II, and III
- 

50. How many continuous real-valued functions  $f$  are there with domain  $[-1, 1]$  such that  $(f(x))^2 = x^2$  for each  $x$  in  $[-1, 1]$ ?

- (A) One
  - (B) Two
  - (C) Three
  - (D) Four
  - (E) Infinitely many
- 

51. Let  $D$  be the region in the  $xy$ -plane in which the series  $\sum_{k=1}^{\infty} \frac{(x+2y)^k}{k}$  converges. Then the interior of  $D$  is

- (A) an open disk
  - (B) the open region bounded by an ellipse
  - (C) the open region bounded by a quadrilateral
  - (D) the open region between two parallel lines
  - (E) an open half plane
- 

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52. Consider the following system of linear equations over the real numbers, where  $x$ ,  $y$ , and  $z$  are variables and  $b$  is a real constant.

$$\begin{aligned}x + y + z &= 0 \\x + 2y + 3z &= 0 \\x + 3y + bz &= 0\end{aligned}$$

Which of the following statements are true?

- I. There exists a value of  $b$  for which the system has no solution.
- II. There exists a value of  $b$  for which the system has exactly one solution.
- III. There exists a value of  $b$  for which the system has more than one solution.

- (A) II only
  - (B) I and II only
  - (C) I and III only
  - (D) II and III only
  - (E) I, II, and III
- 

53. In the complex plane, let  $C$  be the circle  $|z| = 2$  with positive (counterclockwise) orientation. Then

$$\int_C \frac{dz}{(z-1)(z+3)^2} =$$

- (A) 0
  - (B)  $2\pi i$
  - (C)  $\frac{\pi i}{2}$
  - (D)  $\frac{\pi i}{8}$
  - (E)  $\frac{\pi i}{16}$
- 

54. The inside of a certain water tank is a cube measuring 10 feet on each edge and having vertical sides and no top. Let  $h(t)$  denote the water level, in feet, above the floor of the tank at time  $t$  seconds. Starting at time  $t = 0$ , water pours into the tank at a constant rate of 1 cubic foot per second, and simultaneously, water is removed from the tank at a rate of  $0.25h(t)$  cubic feet per second. As  $t \rightarrow \infty$ , what is the limit of the volume of the water in the tank?

- (A) 400 cubic feet
  - (B) 600 cubic feet
  - (C) 1,000 cubic feet
  - (D) The limit does not exist.
  - (E) The limit exists, but it cannot be determined without knowing  $h(0)$ .
- 

GO ON TO THE NEXT PAGE.

55. Suppose that  $f$  is a twice-differentiable function on the set of real numbers and that  $f(0)$ ,  $f'(0)$ , and  $f''(0)$  are all negative. Suppose  $f''$  has all three of the following properties.

- I. It is increasing on the interval  $[0, \infty)$ .
- II. It has a unique zero in the interval  $[0, \infty)$ .
- III. It is unbounded on the interval  $[0, \infty)$ .

Which of the same three properties does  $f$  necessarily have?

- (A) I only
  - (B) II only
  - (C) III only
  - (D) II and III only
  - (E) I, II, and III
- 

56. For every set  $S$  and every metric  $d$  on  $S$ , which of the following is a metric on  $S$ ?

- (A)  $4 + d$
  - (B)  $e^d - 1$
  - (C)  $d - |d|$
  - (D)  $d^2$
  - (E)  $\sqrt{d}$
- 

57. Let  $\mathbb{R}$  be the field of real numbers and  $\mathbb{R}[x]$  the ring of polynomials in  $x$  with coefficients in  $\mathbb{R}$ . Which of the following subsets of  $\mathbb{R}[x]$  is a subring of  $\mathbb{R}[x]$ ?

- I. All polynomials whose coefficient of  $x$  is zero
- II. All polynomials whose degree is an even integer, together with the zero polynomial
- III. All polynomials whose coefficients are rational numbers

- (A) I only
  - (B) II only
  - (C) I and III only
  - (D) II and III only
  - (E) I, II, and III
- 

58. Let  $f$  be a real-valued function defined and continuous on the set of real numbers  $\mathbb{R}$ . Which of the following must be true of the set  $S = \{f(c) : 0 < c < 1\}$ ?

- I.  $S$  is a connected subset of  $\mathbb{R}$ .
- II.  $S$  is an open subset of  $\mathbb{R}$ .
- III.  $S$  is a bounded subset of  $\mathbb{R}$ .

- (A) I only
  - (B) I and II only
  - (C) I and III only
  - (D) II and III only
  - (E) I, II, and III
- 

GO ON TO THE NEXT PAGE.

59. A cyclic group of order 15 has an element  $x$  such that the set  $\{x^3, x^5, x^9\}$  has exactly two elements. The number of elements in the set  $\{x^{13n} : n \text{ is a positive integer}\}$  is

- (A) 3
  - (B) 5
  - (C) 8
  - (D) 15
  - (E) infinite
- 

60. If  $S$  is a ring with the property that  $s = s^2$  for each  $s \in S$ , which of the following must be true?

- I.  $s + s = 0$  for each  $s \in S$ .
- II.  $(s + t)^2 = s^2 + t^2$  for each  $s, t \in S$ .
- III.  $S$  is commutative.

- (A) III only
  - (B) I and II only
  - (C) I and III only
  - (D) II and III only
  - (E) I, II, and III
- 

61. What is the greatest integer that divides  $p^4 - 1$  for every prime number  $p$  greater than 5?

- (A) 12
  - (B) 30
  - (C) 48
  - (D) 120
  - (E) 240
- 

62. The coefficient of  $x^3$  in the expansion of  $(1 + x)^3(2 + x^2)^{10}$  is

- (A)  $2^{14}$
  - (B) 31
  - (C)  $\binom{3}{3} + \binom{10}{1}$
  - (D)  $\binom{3}{3} + 2\binom{10}{1}$
  - (E)  $\binom{3}{3}\binom{10}{1}2^9$
- 

GO ON TO THE NEXT PAGE.

63. At how many points in the  $xy$ -plane do the graphs of  $y = x^{12}$  and  $y = 2^x$  intersect?

- (A) None
  - (B) One
  - (C) Two
  - (D) Three
  - (E) Four
- 

64. Suppose that  $f$  is a continuous real-valued function defined on the closed interval  $[0, 1]$ . Which of the following must be true?

- I. There is a constant  $C > 0$  such that  $|f(x) - f(y)| \leq C$  for all  $x$  and  $y$  in  $[0, 1]$ .
  - II. There is a constant  $D > 0$  such that  $|f(x) - f(y)| \leq 1$  for all  $x$  and  $y$  in  $[0, 1]$  that satisfy  $|x - y| \leq D$ .
  - III. There is a constant  $E > 0$  such that  $|f(x) - f(y)| \leq E|x - y|$  for all  $x$  and  $y$  in  $[0, 1]$ .
- (A) I only
  - (B) III only
  - (C) I and II only
  - (D) II and III only
  - (E) I, II, and III
- 

65. Let  $p(x)$  be the polynomial  $x^3 + ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are real constants. If  $p(-3) = p(2) = 0$  and  $p'(-3) < 0$ , which of the following is a possible value of  $c$ ?

- (A)  $-27$
  - (B)  $-18$
  - (C)  $-6$
  - (D)  $-3$
  - (E)  $-\frac{1}{2}$
- 

66. In the  $xy$ -plane, if  $C$  is the circle  $x^2 + y^2 = 9$ , oriented counterclockwise, then  $\oint_C -2y \, dx + x^2 \, dy = \kappa$

- (A) 0
  - (B)  $6\pi$
  - (C)  $9\pi$
  - (D)  $12\pi$
  - (E)  $18\pi$
- 

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST.

NOTE: To ensure prompt processing of test results, it is important that you fill in the blanks exactly as directed.

### SUBJECT TEST

A. Print and sign your full name in this box:

PRINT:	(LAST)	(FIRST)	(MIDDLE)
SIGN:			

Copy this code in box 6 on your answer sheet. Then fill in the corresponding ovals exactly as shown.

6. TITLE CODE				
6	8	7	5	0
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
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Copy the Test Name and Form Code in box 7 on your answer sheet.

TEST NAME Mathematics

FORM CODE GR9768

### GRADUATE RECORD EXAMINATIONS SUBJECT TEST

B. The Subject Tests are intended to measure your achievement in a specialized field of study. Most of the questions are concerned with subject matter that is probably familiar to you, but some of the questions may refer to areas that you have not studied.

Your score will be determined by subtracting one-fourth the number of incorrect answers from the number of correct answers. Questions for which you mark no answer or more than one answer are not counted in scoring. If you have some knowledge of a question and are able to rule out one or more of the answer choices as incorrect, your chances of selecting the correct answer are improved, and answering such questions will likely improve your score. It is unlikely that pure guessing will raise your score; it may lower your score.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult for you. Go on to the other questions and come back to the difficult ones later if you can.

YOU MUST INDICATE ALL YOUR ANSWERS ON THE SEPARATE ANSWER SHEET. No credit will be given for anything written in this examination book, but you may write in the book as much as you wish to work out your answers. After you have decided on your response to a question, fill in the corresponding oval on the answer sheet. BE SURE THAT EACH MARK IS DARK AND COMPLETELY FILLS THE OVAL. Mark only one answer to each question. No credit will be given for multiple answers. Erase all stray marks. If you change an answer, be sure that all previous marks are erased completely. Incomplete erasures may be read as intended answers. Do not be concerned that the answer sheet provides spaces for more answers than there are questions in the test.

Example:

What city is the capital of France?

- (A) Rome
- (B) Paris
- (C) London
- (D) Cairo
- (E) Oslo

Sample Answer

(A)  (C) (D) (E)

CORRECT ANSWER  
PROPERLY MARKED

(A)  (C) (D) (E)

(A)  (C) (D) (E)

(A)  (C) (D) (E)

(A)  (C) (D) (E)

IMPROPER MARKS

**DO NOT OPEN YOUR TEST BOOK UNTIL YOU ARE TOLD TO DO SO.**



## Scoring Your Subject Test

The GRE Mathematics Test (Rescaled) scores typically range from 440 to 810. The range for different editions of a given test may vary because different editions are not of precisely the same difficulty. The differences in ranges among different editions of a given test, however, usually are small. This should be taken into account, especially when comparing two very high scores. In general, differences between scores at the 99th percentile should be ignored. **The score conversion table on page 57 shows the score range for this edition of the test only.**

The worksheet on page 56 lists the correct answers to the questions. Columns are provided for you to mark whether you chose the correct (C) answer or an incorrect (I) answer to each question. Draw a line across any question you omitted, because it is not counted in the scoring. At the bottom of the page, enter the total number correct and the total number incorrect. Divide the total incorrect by 4 and subtract the resulting number from the total correct. This is the adjustment made for guessing. Then round the result to the nearest whole number. This will give you your raw total score. Use the total score conversion table to find the scaled total score that corresponds to your raw total score.

Example: Suppose you chose the correct answers to 48 questions and incorrect answers to 15. Dividing 15 by 4 yields 3.75. Subtracting 3.75 from 48 equals 44.25, which is rounded to 44. The raw score of 44 corresponds to a scaled score of 710 on the new GRE Mathematics scale, effective October 2001. The scaled scores in this book, and those received by individuals taking the GRE Mathematics Test (Rescaled) after October 2001, are **NOT** comparable to Mathematics Test scores reported prior to October 2001.



DO NOT USE INK

Use only a pencil with soft, black lead (No. 2 or HB) to complete this answer sheet. Be sure to fill in completely the space that corresponds to your answer choice. Completely erase any errors or stray marks.

1. NAME Enter your last name, first name initial (given name), and middle initial if you have one. Omit spaces, apostrophes, Jr., II, etc.

Name entry grid with bubbles for letters and initials.

2. YOUR NAME: Last Name (Family or Surname) First Name (Given) M.I.

MAILING ADDRESS: P.O. Box or Street Address

City State or Province

Country Zip or Postal Code

CENTER: City State or Province

Country Center Number Room Number

SIGNATURE:

GRADUATE RECORD EXAMINATIONS - GRE - SUBJECT TEST

SIDE 1

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

Main answer grid with 38 rows and multiple columns of bubbles for letters A-E.

3. DATE OF BIRTH

Month, Day, Year selection bubbles.

4. SOCIAL SECURITY NUMBER (U.S.A. only)

Social Security Number selection bubbles.

5. REGISTRATION NUMBER (from your admission ticket)

Registration Number selection bubbles.

6. TITLE CODE (on back cover of your test book)

Title Code selection bubbles.

7. TEST NAME (on back cover of your test book) FORM CODE (on back cover of your test book)

Test Name and Form Code selection bubbles.

8. TEST BOOK SERIAL NUMBER (see number in upper right corner of front cover of your test book)

Test Book Serial Number selection bubbles.

SHADED AREA FOR ETS USE ONLY

SIDE 2

SUBJECT TEST

COMPLETE THE  
CERTIFICATION STATEMENT,  
THEN TURN ANSWER SHEET  
OVER TO SIDE 1.

CERTIFICATION STATEMENT

Please write the following statement below, DO NOT PRINT.

"I certify that I am the person whose name appears on this answer sheet. I also agree not to disclose the contents of the test I am taking today to anyone." Sign and date where indicated.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

SIGNATURE: \_\_\_\_\_ DATE: \_\_\_\_\_  
Month Day Year

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ●  
YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

115	A	B	C	D	E	147	A	B	C	D	E	179	A	B	C	D	E	211	A	B	C	D	E
116	A	B	C	D	E	148	A	B	C	D	E	180	A	B	C	D	E	212	A	B	C	D	E
117	A	B	C	D	E	149	A	B	C	D	E	181	A	B	C	D	E	213	A	B	C	D	E
118	A	B	C	D	E	150	A	B	C	D	E	182	A	B	C	D	E	214	A	B	C	D	E
119	A	B	C	D	E	151	A	B	C	D	E	183	A	B	C	D	E	215	A	B	C	D	E
120	A	B	C	D	E	152	A	B	C	D	E	184	A	B	C	D	E	216	A	B	C	D	E
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122	A	B	C	D	E	154	A	B	C	D	E	186	A	B	C	D	E	218	A	B	C	D	E
123	A	B	C	D	E	155	A	B	C	D	E	187	A	B	C	D	E	219	A	B	C	D	E
124	A	B	C	D	E	156	A	B	C	D	E	188	A	B	C	D	E	220	A	B	C	D	E
125	A	B	C	D	E	157	A	B	C	D	E	189	A	B	C	D	E	221	A	B	C	D	E
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130	A	B	C	D	E	162	A	B	C	D	E	194	A	B	C	D	E	226	A	B	C	D	E
131	A	B	C	D	E	163	A	B	C	D	E	195	A	B	C	D	E	227	A	B	C	D	E
132	A	B	C	D	E	164	A	B	C	D	E	196	A	B	C	D	E	228	A	B	C	D	E
133	A	B	C	D	E	165	A	B	C	D	E	197	A	B	C	D	E	229	A	B	C	D	E
134	A	B	C	D	E	166	A	B	C	D	E	198	A	B	C	D	E	230	A	B	C	D	E
135	A	B	C	D	E	167	A	B	C	D	E	199	A	B	C	D	E	231	A	B	C	D	E
136	A	B	C	D	E	168	A	B	C	D	E	200	A	B	C	D	E	232	A	B	C	D	E
137	A	B	C	D	E	169	A	B	C	D	E	201	A	B	C	D	E	233	A	B	C	D	E
138	A	B	C	D	E	170	A	B	C	D	E	202	A	B	C	D	E	234	A	B	C	D	E
139	A	B	C	D	E	171	A	B	C	D	E	203	A	B	C	D	E	235	A	B	C	D	E
140	A	B	C	D	E	172	A	B	C	D	E	204	A	B	C	D	E	236	A	B	C	D	E
141	A	B	C	D	E	173	A	B	C	D	E	205	A	B	C	D	E	237	A	B	C	D	E
142	A	B	C	D	E	174	A	B	C	D	E	206	A	B	C	D	E	238	A	B	C	D	E
143	A	B	C	D	E	175	A	B	C	D	E	207	A	B	C	D	E	239	A	B	C	D	E
144	A	B	C	D	E	176	A	B	C	D	E	208	A	B	C	D	E	240	A	B	C	D	E
145	A	B	C	D	E	177	A	B	C	D	E	209	A	B	C	D	E	241	A	B	C	D	E
146	A	B	C	D	E	178	A	B	C	D	E	210	A	B	C	D	E	242	A	B	C	D	E

IF YOU DO NOT WANT THIS ANSWER SHEET TO BE SCORED

If you want to cancel your scores from this administration, complete A and B below. You will not receive scores for this test; however, you will receive confirmation of this cancellation. No record of this test or the cancellation will be sent to the recipients you indicated, and there will be no scores for this test on your GRE file. Once a score is canceled, it cannot be reinstated.

To cancel your scores from this test administration, you must:

A. fill in both ovals here . . .   B. sign your full name here: \_\_\_\_\_

TR	TW	TFS	TCS	1R	1W	1FS	1CS	2R	2W	2FS	2CS
FOR ETS USE ONLY				3R	3W	3FS	3CS	4R	4W	4FS	4CS
				5R	5W	5FS	5CS	6R	6W	6FS	6CS