Test 8767

MATHEMATICS TEST

Time—170 minutes

66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is the best of the choices offered and then mark the corresponding space on the answer sheet.

Computation and scratchwork may be done in this examination book.

Note: In this examination:

(1) All logarithms are to the base e unless otherwise specified.
(2) The set of all x such that $a \leqq x \leqq b$ is denoted by $[a, b]$.

1. If S is a plane in Euclidean 3-space containing $(0, 0, 0)$, $(2, 0, 0)$, and $(0, 0, 1)$, then S is the

(A) xy-plane
(B) xz-plane
(C) yz-plane
(D) plane $y - z = 0$
(E) plane $x + 2y - 2z = 0$

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2. If a, b, and c are real numbers, which of the following are necessarily true?

I. If $a < b$ and $ab \neq 0$, then $\frac{1}{a} > \frac{1}{b}$.

II. If $a < b$, then $ac < bc$ for all c.

III. If $a < b$, then $a + c < b + c$ for all c.

IV. If $a < b$, then $-a > -b$.

(A) I only (B) I and III only (C) III and IV only (D) II, III, and IV only (E) I, II, III, and IV

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3. $\int_0^1 \int_0^x xy\,dy\,dx =$

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{3}$ (D) 1 (E) 3

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4. For $x \geqq 0$, $\frac{d}{dx}(x^e \cdot e^x) =$

(A) $x^e \cdot e^x + x^{e-1} \cdot e^{x+1}$ (B) $x^e \cdot e^x + x^{e+1} \cdot e^{x-1}$ (C) $x^e \cdot e^x$ (D) $x^{e-1} \cdot e^{x+1}$ (E) $x^{e+1} \cdot e^{x-1}$

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5. All functions f defined on the xy-plane such that

$$\frac{\partial f}{\partial x} = 2x + y \quad \text{and} \quad \frac{\partial f}{\partial y} = x + 2y$$

are given by $f(x, y) =$

(A) $x^2 + xy + y^2 + C$ (B) $x^2 - xy + y^2 + C$ (C) $x^2 - xy - y^2 + C$

(D) $x^2 + 2xy + y^2 + C$ (E) $x^2 - 2xy + y^2 + C$

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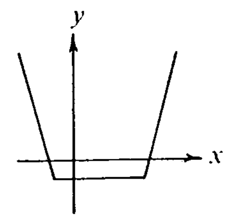
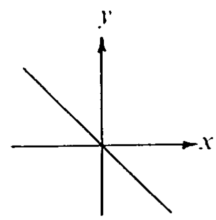


6. Which of the following could be the graph of the derivative of the function whose graph is shown in the figure above?

(A)



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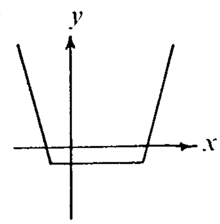

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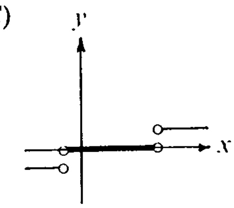

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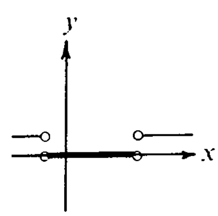

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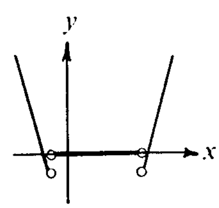

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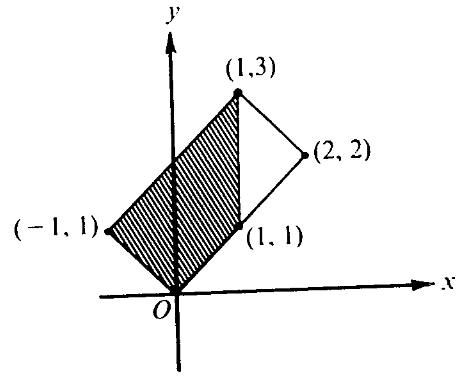

7. Which of the following integrals represents the area of the shaded portion of the rectangle shown in the figure above?

(A) $\int_{-1}^{1} (x + 2 - |x|)\,dx$ (B) $\int_{-1}^{1} (|x| + x + 2)\,dx$ (C) $\int_{-1}^{1} (x + 2)\,dx$

(D) $\int_{-1}^{1} |x|\,dx$ (E) $\int_{-1}^{1} 2\,dx$

8. $\sum_{n=1}^{\infty} \frac{n}{n+1} =$

(A) $\frac{1}{e}$ (B) $\log 2$ (C) 1 (D) e (E) $+\infty$

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9. k digits are to be chosen at random (with repetitions allowed) from $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the probability that 0 will not be chosen?

(A) $\frac{1}{k}$ (B) $\frac{1}{10}$ (C) $\frac{k-1}{k}$ (D) $\left(\frac{1}{10}\right)^k$ (E) $\left(\frac{9}{10}\right)^k$

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10. In order to send an undetected message to an agent in the field, each letter in the message is replaced by the number of its position in the alphabet and that number is entered in a matrix M. Thus, for example, "DEAD" becomes the matrix $M = \begin{pmatrix} 4 & 5 \\ 1 & 4 \end{pmatrix}$. In order to further avoid detection, each message with four letters is sent to the agent encoded as MC, where $C = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$. If the agent receives the matrix $\begin{pmatrix} 51 & -3 \\ 31 & -8 \end{pmatrix}$, then the message

(A) RUSH (B) COME (C) ROME (D) CALL

(E) not uniquely determined by the information given

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11. If $\sin^{-1}x = \frac{\pi}{6}$, then the acute angle value of $\cos^{-1}x$ is

(A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\sqrt{1 - \frac{\pi^2}{6^2}}$ (D) $1 - \frac{\pi}{6}$ (E) 0

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12. $\int_0^{\pi} e^{\sin^2 x} e^{\cos^2 x} dx =$

(A) π (B) $e\pi$ (C) e^{π} (D) $e^{\sin^2 \pi}$ (E) $e^{\pi} - 1$

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13. Which of the following is true of the behavior of $f(x) = \dfrac{x^3 + 8}{x^2 - 4}$ as $x \to 2$?

(A) The limit is 0.
(B) The limit is 1.
(C) The limit is 4.
(D) The graph of the function has a vertical asymptote at 2.
(E) The function has unequal, finite left-hand and right-hand limits.

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14. A newscast contained the statement that the total use of electricity in city A had declined in one billing period by 5 percent, while household use had declined by 4 percent and all other uses increased by 25 percent. Which of the following must be true about the billing period?

(A) The statement was in error.

(B) The ratio of all other uses to household use was $\frac{29}{1}$.

(C) The ratio of all other uses to household use was $\frac{29}{16}$.

(D) The ratio of all other uses to household use was $\frac{29}{19}$.

(E) None of the above

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15. If f is a linear transformation from the plane to the real numbers and if $f(1, 1) = 1$ and $f(-1, 0) = 2$, then $f(3, 5) =$

(A) -6 (B) -5 (C) 0 (D) 8 (E) 9

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16. Suppose that an arrow is shot from a point p and lands at a point q such that at one and only one point in its flight is the arrow parallel to the line of sight between p and q. Of the following, which is the best mathematical model for the phenomenon described above?

(A) A function f differentiable on $[a, b]$ such that there is one and only one point c in $[a, b]$ with $\int_a^b f'(x)\,dx = c(b - a)$

(B) A function f whose second derivative is at all points negative such that there is one and only one point c in $[a, b]$ with $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

(C) A function f whose first derivative is at all points positive such that there is one and only one point c in $[a, b]$ with $\int_a^b f(x)\,dx = f(c) \cdot (b - a)$

(D) A function f continuous on $[a, b]$ such that there is one and only one point c in $[a, b]$ with $\int_a^b f(x)\,dx = f(c) \cdot (b - a)$

(E) A function f continuous on $[a, b]$ and $f(a) < d < f(b)$ such that there is one and only one point c in $[a, b]$ with $f(c) = d$

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17. Let $*$ be the binary operation on the rational numbers given by $a * b = a + b + 2ab$. Which of the following are true?

I. $*$ is commutative.
II. There is a rational number that is a $*$-identity.
III. Every rational number has a $*$-inverse.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

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18. A group G in which $(ab)^2 = a^2b^2$ for all a, b in G is necessarily

(A) finite
(B) cyclic
(C) of order two
(D) abelian
(E) none of the above

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19. If $c > 0$ and $f(x) = e^x - cx$ for all real numbers x, then the minimum value of f is

(A) $f(c)$ (B) $f(e^c)$ (C) $f\left(\frac{1}{c}\right)$ (D) $f(\log c)$ (E) nonexistent

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20. Suppose that $f(1 + x) = f(x)$ for all real x. If f is a polynomial and $f(5) = 11$, then $f\left(\frac{15}{2}\right)$ is

(A) -11 (B) 0 (C) 11 (D) $\frac{33}{2}$

(E) not uniquely determined by the information given

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21. For all $x > 0$, if $f(\log x) = \sqrt{x}$, then $f(x) =$

(A) $e^{\frac{x}{2}}$ (B) $\log\sqrt{x}$ (C) $e^{\sqrt{x}}$ (D) $\sqrt{\log x}$ (E) $\frac{\log x}{2}$

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22. $\int_0^1 \left(\int_0^{\sin y} \frac{1}{\sqrt{1 - x^2}}\, dx \right) dy =$

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{4}$ (D) 1 (E) $\frac{\pi}{3}$

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23. $S(n)$ is a statement about positive integers n such that whenever $S(k)$ is true, $S(k+1)$ must also be true. Furthermore, there exists some positive integer n_0 such that $S(n_0)$ is not true. Of the following, which is the strongest conclusion that can be drawn?

(A) $S(n_0 + 1)$ is not true.
(B) $S(n_0 - 1)$ is not true.
(C) $S(n)$ is not true for any $n \leqq n_0$.
(D) $S(n)$ is not true for any $n \geqq n_0$.
(E) $S(n)$ is not true for any n.

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24. Let f and g be functions defined on the positive integers and related in the following way:

$$f(n) = \begin{cases} 1, & \text{if } n = 1 \\ 2f(n-1), & \text{if } n \neq 1 \end{cases}$$

and

$$g(n) = \begin{cases} 3g(n+1), & \text{if } n \neq 3 \\ f(n), & \text{if } n = 3. \end{cases}$$

The value of $g(1)$ is

(A) 6 (B) 9 (C) 12 (D) 36
(E) not uniquely determined by the information given

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25. Let x and y be positive integers such that $3x + 7y$ is divisible by 11. Which of the following must also be divisible by 11 ?

(A) $4x + 6y$ (B) $x + y + 5$ (C) $9x + 4y$ (D) $4x - 9y$ (E) $x + y - 1$

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26. If k is a real number and

$$f(x) = \begin{cases} \sin \frac{1}{x} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases}$$

and if the graph of f is not a connected subset of the plane, then the value of k

(A) could be -1
(B) must be 0
(C) must be 1
(D) could be less than 1 and greater than -1
(E) must be less than -1 or greater than 1

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27. For what triples of real numbers (a, b, c) with $a \neq 0$ is the function

defined by $f(x) = \begin{cases} x, & \text{if } x \leqq 1 \\ ax^2 + bx + c, & \text{if } x > 1 \end{cases}$

differentiable at all real x?

(A) $\{(a,\ 1 - 2a,\ a) \mid a$ is a nonzero real number$\}$

(B) $\{(a,\ 1 - 2a,\ c) \mid a, c$ are real numbers and $a \neq 0\}$

(C) $\{(a, b, c) \mid a, b, c$ are real numbers, $a \neq 0$, and $a + b + c = 1\}$

(D) $\left\{\left(\frac{1}{2}, 0, 0\right)\right\}$

(E) $\{(a,\ 1 - 2a,\ 0) \mid a$ is a nonzero real number$\}$

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Questions 28-30 are based on the following information.

Let f be a function such that the graph of f is a semicircle S with end points $(a, 0)$ and $(b, 0)$ where $a < b$.

28. $\left|\int_a^b f(x)dx\right| =$

(A) $f(b) - f(a)$ (B) $\frac{f(b) - f(a)}{b - a}$ (C) $(b - a)\frac{\pi}{4}$ (D) $(b - a)^2\pi$ (E) $(b - a)^2\frac{\pi}{8}$

29. The graph of $y = 3f(x)$ is a

(A) translation of S (B) semicircle with radius three times that of S (C) subset of an ellipse
(D) subset of a parabola (E) subset of a hyperbola

30. The improper integral $\int_a^b f(x)f'(x)dx$ is

(A) necessarily zero
(B) possibly zero but not necessarily
(C) necessarily nonexistent
(D) possibly nonexistent but not necessarily
(E) none of the above

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31. $\lim_{x \to \pi} \dfrac{e^{-\pi} - e^{-x}}{\sin x} =$

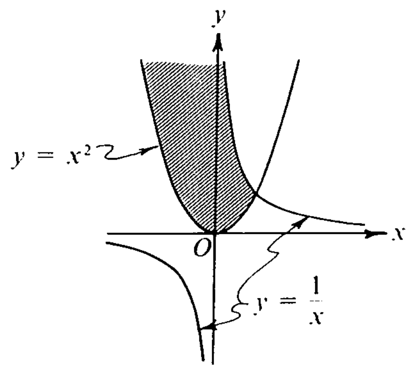
(A) $-\infty$ (B) $-e^{-\pi}$ (C) 0 (D) $e^{-\pi}$ (E) 1

---

32. The dimension of the subspace spanned by the real vectors

$$\begin{pmatrix}1\\1\\0\\0\end{pmatrix}, \begin{pmatrix}2\\2\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}2\\0\\0\\3\end{pmatrix}, \begin{pmatrix}1\\-2\\0\\8\end{pmatrix}, \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \text{ is}$$

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6



33. The shaded region in the figure above indicates the graph of which of the following?

(A) $x^2 < y$ and $y < \frac{1}{x}$ (B) $x^2 < y$ or $y < \frac{1}{x}$ (C) $x^2 > y$ and $y > \frac{1}{x}$

(D) $x^2 > y$ or $y > \frac{1}{x}$ (E) $x^2 < y$ and $xy < 1$

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34. Let the bottom edge of a rectangular mirror on a vertical wall be parallel to and h feet above the level floor. If a person with eyes t feet above the floor is standing erect at a distance d feet from the mirror, what is the relationship among h, d, and t if the person can just see his own feet in the mirror?

(A) $t = 2h$ and d does not matter.
(B) $t = 4d$ and h does not matter.
(C) $h^2 + d^2 = \frac{t^2}{4}$
(D) $t - h = d$
(E) $(t - h)^2 = 4d$

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35. The rank of the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{pmatrix} \text{ is}$$

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

---

36. The shortest distance from the curve $xy = 8$ to the origin is

(A) 4
(B) 8
(C) 16
(D) $2\sqrt{2}$
(E) $4\sqrt{2}$

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37. What is wrong with the following argument?

Let R be the real numbers.

(1) "For all $x,\ y \in R,\ f(x) + f(y) = f(xy)$."

is equivalent to

(2) "For all $x,\ y \in R,\ f(-x) + f(y) = f((-x)y)$."

which is equivalent to

(3) "For all $x,\ y \in R,\ f(-x) + f(y) = f((-x)y) = f(x(-y)) = f(x) + f(-y)$."

From this for $y = 0$, we make the conclusion

(4) "For all $x \in R,\ f(-x) = f(x)$."

Since the steps are reversible, any function with property (4) has property (1). Therefore, for all $x,\ y \in R,\ \cos x + \cos y = \cos(xy)$.

(A) (2) does not imply (1).

(B) (3) does not imply (2).

(C) (3) does not imply (4).

(D) (4) does not imply (3).

(E) (4) is not true for $f = \cos$.

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38. If M is the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, then M^{100} is

(A) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(E) none of the above

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39. If $f(x) = \begin{cases} \dfrac{|x|}{x}, & \text{for } x \neq 0 \\ 0, & \text{for } x = 0, \end{cases}$ then $\int_{-1}^{1} f(x)\,dx$ is

(A) -2 (B) 0 (C) 2 (D) not defined
(E) none of the above

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40. Let $y = f(x)$ be a solution of the differential equation $x\,dy + (y - xe^x)\,dx = 0$ such that $y = 0$ when $x = 1$. What is the value of $f(2)$?

(A) $\frac{1}{2e}$ (B) $\frac{1}{e}$ (C) $\frac{e^2}{2}$ (D) $2e$ (E) $2e^2$

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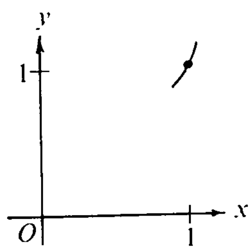
41. Of the following, which best represents a portion of the graph of $y = \frac{1}{e^x} + x - \frac{1}{e}$ near (1, 1) ?

(A)


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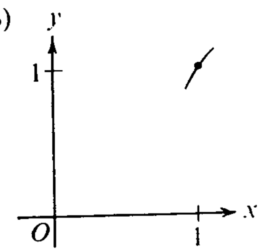

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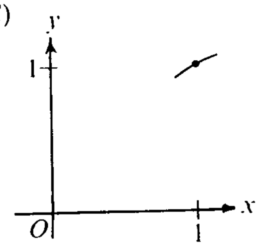

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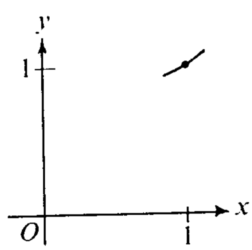

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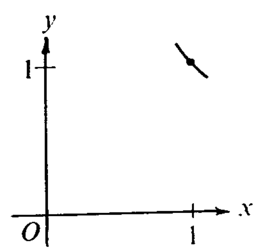

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42. In xyz-space, the degree measure of the angle between the rays

$$z = x \geqq 0,\ y = 0$$

and

$$z = y \geqq 0,\ x = 0 \quad \text{is}$$

(A) 0° (B) 30° (C) 45° (D) 60° (E) 90°

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43. If a polynomial $f(x)$ over the real numbers has the complex numbers $2 + i$ and $1 - i$ as roots, then $f(x)$ could be

(A) $x^4 + 6x^3 + 10$
(B) $x^4 + 7x^2 + 10$
(C) $x^3 - x^2 + 4x + 1$
(D) $x^3 + 5x^2 + 4x + 1$
(E) $x^4 - 6x^3 + 15x^2 - 18x + 10$

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44. Suppose f is a real function such that $f'(x_0)$ exists. Which of the following is the value of

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0 - h)}{h}\ ?$$

(A) 0 (B) $2f'(x_0)$ (C) $f'(-x_0)$ (D) $-f'(x_0)$ (E) $-2f'(x_0)$

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45. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{e^n}{n!} x^n$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) $+\infty$

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46. In the xy-plane, the graph of $x^{\log y} = y^{\log x}$ is

(A) empty
(B) a single point
(C) a ray in the open first quadrant
(D) a closed curve
(E) the open first quadrant

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47. Suppose that the space S contains exactly eight points. If $\mathcal{G}$ is a collection of 250 distinct subsets of S, which of the following statements must be true?

(A) S is an element of $\mathcal{G}$.

(B) $\bigcap_{G \in \mathcal{G}} G = S$

(C) $\bigcap_{G \in \mathcal{G}} G$ is a nonempty proper subset of S.

(D) $\mathcal{G}$ has a member that contains exactly one element.

(E) The empty set is an element of $\mathcal{G}$.

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48. Let V be the set of all real polynomials $p(x)$. Let transformations T, S be defined on V by $T: p(x) \to xp(x)$ and $S: p(x) \to p'(x) = \frac{d}{dx}p(x)$, and interpret $(ST)(p(x))$ as $S(T(p(x)))$. Which of the following is true?

(A) $ST = 0$
(B) $ST = T$
(C) $ST = TS$
(D) $ST - TS$ is the identity map of V onto itself.
(E) $ST + TS$ is the identity map of V onto itself.

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49. If the finite group G contains a subgroup of order seven but no element (other than the identity) is its own inverse, then the order of G could be

(A) 27 (B) 28 (C) 35 (D) 37 (E) 42

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50. In a game two players take turns tossing a fair coin; the winner is the first one to toss a head. The probability that the player who makes the first toss wins the game is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

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51. Let $x_1 = 1$ and $x_{n+1} = \sqrt{3 + 2x_n}$ for all positive integers n. If it is assumed that $\{x_n\}$ converges, then $\lim_{n \to \infty} x_n =$

(A) -1 (B) 0 (C) $\sqrt{5}$ (D) e (E) 3

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52. Which of the following is the larger of the eigenvalues (characteristic values) of the matrix $\begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$?

(A) 4 (B) 5 (C) 6 (D) 10 (E) 12

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53. Let V be the vector space, under the usual operations, of real polynomials that are of degree at most 3. Let W be the subspace of all polynomials $p(x)$ in V such that $p(0) = p(1) = p(-1) = 0$. Then $\dim V + \dim W$ is

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

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54. The map $x \to axa^2$ of a group G into itself is a homomorphism if and only if

(A) G is abelian (B) $G = \{e\}$ (C) $a = e$ (D) $a^2 = a$ (E) $a^3 = e$

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55. Let $f(x, y) = x^3 + y^3 + 3xy$ for all real x and y. Then there exist distinct points P and Q such that f has a

(A) local maximum at P and at Q
(B) saddle point at P and at Q
(C) local maximum at P and a saddle point at Q
(D) local minimum at P and a saddle point at Q
(E) local minimum at P and at Q

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56. The polynomial $p(x) = 1 + \frac{1}{2}(x - 1) - \frac{1}{8}(x - 1)^2$ is used to approximate $\sqrt{1.01}$. Which of the following most closely approximates the error $\sqrt{1.01} - p(1.01)$?

(A) $\left(\frac{1}{16}\right) \times 10^{-6}$

(B) $\left(\frac{1}{48}\right) \times 10^{-8}$

(C) $\left(\frac{3}{8}\right) \times 10^{-10}$

(D) $-\left(\frac{3}{8}\right) \times 10^{-10}$

(E) $-\left(\frac{1}{16}\right) \times 10^{-6}$

---

57. Acceptable input for a certain pocket calculator is a finite sequence of characters each of which is either a digit or a sign. The first character must be a digit, the last character must be a digit, and any character that is a sign must be followed by a digit. There are 10 possible digits and 4 possible signs. If N_k denotes the number of such acceptable sequences having length k, then N_k is given recursively by

(A) $N_1 = 10$
$N_k = 10N_{k-1}$

(B) $N_1 = 10$
$N_k = 14N_{k-1}$

(C) $N_1 = 10$
$N_2 = 100$
$N_k = 10N_{k-1} + 40N_{k-2}$

(D) $N_1 = 10$
$N_2 = 140$
$N_k = 14N_{k-1} + 40N_{k-2}$

(E) $N_1 = 14$
$N_2 = 196$
$N_k = 10N_{k-1} + 14N_{k-2}$

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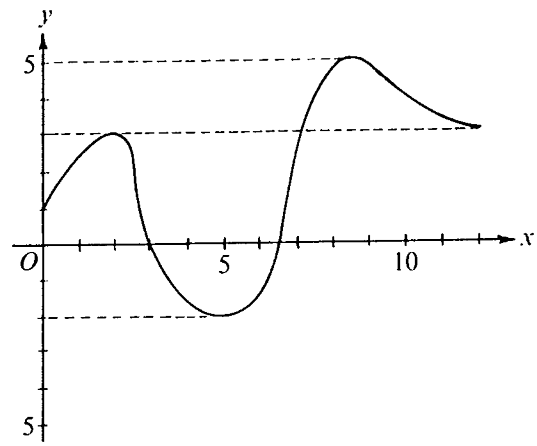
58. If $f(z)$ is an analytic function that maps the entire finite complex plane into the real axis, then the imaginary axis must be mapped onto

(A) the entire real axis
(B) a point
(C) a ray
(D) an open finite interval
(E) the empty set



59. If f is the function whose graph is indicated in the figure above, then the least upper bound (supremum) of

$$\left\{ \sum_{k=1}^{n} |f(x_k) - f(x_{k-1})| : 0 = x_0 < x_1 < \cdots < x_{n-1} < x_n = 12 \right\}$$

appears to be

(A) 2 (B) 7 (C) 12 (D) 16 (E) 21

---

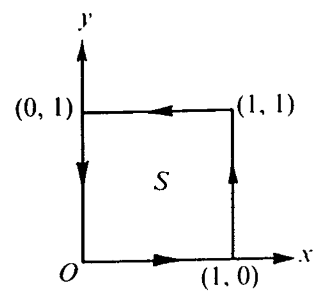
60. A fair die is tossed 360 times. The probability that a six comes up on 70 or more of the tosses is

(A) greater than 0.50
(B) between 0.16 and 0.50
(C) between 0.02 and 0.16
(D) between 0.01 and 0.02
(E) less than 0.01

GO ON TO THE NEXT PAGE.

61. Let $I \neq A \neq -I$, where I is the identity matrix and A is a real 2×2 matrix. If $A = A^{-1}$, then the trace of A is

(A) 2 (B) 1 (C) 0 (D) -1 (E) -2



62. If B is the boundary of S as indicated in the figure above, then $\int_B (3y\,dx + 4x\,dy) =$

(A) 0 (B) 1 (C) 3 (D) 4 (E) 7

63. Let f be a continuous, strictly decreasing, real-valued function such that $\int_0^{+\infty} f(x)\,dx$ is finite and $f(0) = 1$. In terms of f^{-1} (the inverse function of f), $\int_0^{+\infty} f(x)\,dx$ is

(A) less than $\int_1^{+\infty} f^{-1}(y)\,dy$

(B) greater than $\int_0^1 f^{-1}(y)\,dy$

(C) equal to $\int_1^{+\infty} f^{-1}(y)\,dy$

(D) equal to $\int_0^1 f^{-1}(y)\,dy$

(E) equal to $\int_0^{+\infty} f^{-1}(y)\,dy$

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64. Let S be a compact topological space, let T be a topological space, and let f be a function from S onto T. Of the following conditions on f, which is the weakest condition sufficient to ensure the compactness of T?

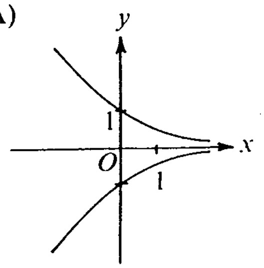
(A) f is a homeomorphism.
(B) f is continuous and $1-1$.
(C) f is continuous.
(D) f is $1-1$.
(E) f is bounded.

---

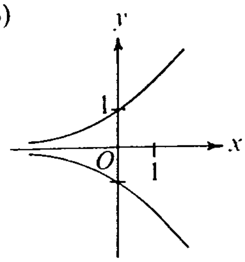
65. Which of the following indicates the graphs of two functions that satisfy the differential equation

$$\left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx} + y^2 = 0\,?$$

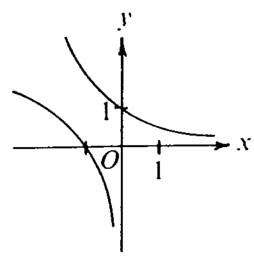
(A)



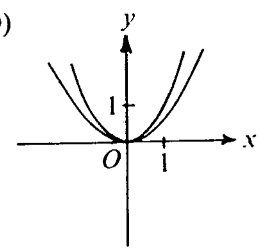
(B)



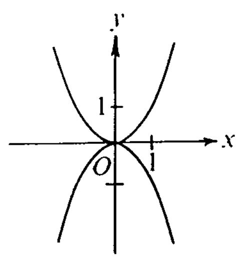
(C)



(D)



(E)



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66. Which of the following subsets are subrings of the ring of real numbers?

I. $\{a + b\sqrt{2} \mid a$ and b are rational$\}$

II. $\left\{\frac{n}{3^m} \mid n \text{ is an integer and } m \text{ is a non-negative integer}\right\}$

III. $\{a + b\sqrt{5} \mid a$ and b are real numbers and $a^2 + b^2 \leq 1\}$

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST.

WORK SHEET for the MATHEMATICS Test, Form GR8767 ONLY

Answer Key and Percentage* of Examinees Answering Each Question Correctly

QUESTION Number	QUESTION Answer	P+	TOTAL C	TOTAL I
1	B	92		
2	C	72		
3	B	94		
4	A	89		
5	A	89		
6	C	83		
7	A	81		
8	E	76		
9	E	84		
10	C	79		
11	B	77		
12	B	81		
13	D	82		
14	A	47		
15	E	77		
16	B	61		
17	C	49		
18	D	65		
19	D	71		
20	C	42		
21	A	64		
22	B	54		
23	C	56		
24	D	80		
25	D	53		
26	E	54		
27	A	34		
28	E	78		
29	C	58		
30	A	29		
31	B	58		
32	B	62		
33	E	41		
34	A	51		
35	B	29		
36	A	54		
37	D	38		
38	A	69		
39	B	63		
40	C	30		

Correct (C) ________

Incorrect (I) ________

QUESTION Number	QUESTION Answer	P+	TOTAL C	TOTAL I
41	D	47		
42	D	33		
43	E	49		
44	B	57		
45	E	46		
46	E	42		
47	D	48		
48	D	67		
49	C	41		
50	D	40		
51	E	52		
52	C	59		
53	B	23		
54	E	39		
55	C	16		
56	A	31		
57	C	46		
58	B	37		
59	D	35		
60	C	23		
61	C	37		
62	B	33		
63	D	40		
64	C	39		
65	A	48		
66	B	57		

Correct (C) ________

Incorrect (I) ________

Total Score

C − I/4 = ________

Scaled Score (SS) = ________

*Estimated P+ for the group of examinees who took the GRE Mathematics Test in a recent three-year period.